N. C. Schumacher

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EXPERIMENTAL DETERMINATION

OF RESIDUAL STRESSES IN

AN ORTHOTROPIC MATERIAL BY THE USE OF

STRAIN GAGES AND MATERIAL REMOVAL

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MISSILE & SPACE SYSTEMS DIVISION
DOUGLAS AIRCRAFT COMPANY, INC.
SANTA MONICA/CALIFORNIA

DOUGLAS

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ABSTRACT

A simple, accurate, computerized technique has been developed, and experimentally verified, to compute, print, plot and draw a curve of the residual stress distribution in two orthogonal directions of an isotropic or orthotropic structure.

Strain gage measurements are made on the two opposite surfaces of an element before and after being cut from the structure, and linear stresses are computed from the values of the released strains. The non-linear residual stress distribution across the element is determined by removing layers of material from one side. Thickness and strain readings are taken before and after removal of each layer. These data plus Poisson's ratios and the elastic moduli in two orthogonal directions become the input to the Fortran IV program, which computes, tabulates, plots and draws a curve of the linear and non-linear residual stresses.

The required minimum length and width of the specimen in relation to the thickness has been experimentally determined.

A literature survey of the various techniques which have been used in the past to determine residual stresses is included with comments as to their advantages and disadvantages.

DESCRIPTORS

Residual Stress
Stress
Initial Stress
Material Removal
Layer Removal
Strain Gage
Photostress
Orthotropic
Experimental

PREFACE

Residual stresses have long been suspected as being one of the factors influencing premature failure of some structural elements.

It therefore becomes mandatory for the structural engineer to know the magnitude of these stresses.

This document has been written to outline a simple, accurate, computerized technique which has been developed to determine and display the residual stresses in two orthogonal directions of an isotropic or orthotropic structure.

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1. INTRODUCTION

A simple, accurate, computerized technique has been developed, and experimentally verified, to compute, print, plot and draw a curve of the residual stress distribution in two orthogonal directions of an isotropic or orthotropic structure.

1.1 Types and Causes of Residual Stresses

Residual stresses may be classified as linear and non-linear. Linear residual stresses are the result of moment and/or axial load applied to an element of a structure by its adjacent elements. An example of this type of residual stress is a flat sheet which is elastically deformed in the process of assembly to fit around circular frames thus inducing a linear residual stress in the sheet which is due to moment. Non-linear residual stresses are the stresses which exist in an element after it has been removed from its adjacent elements. Since the element is in equilibrium, the sum of the forces and moments at any cross-section are equal to zero. The stress distribution across the section is characterized by the fact that it is non-linear, i.e., it does not vary uniformly from one surface to the other surface. An example of this type of residual stress is that which is introduced into a bar when it is bent beyond the proportional limit and subsequently released. Welding, shot-peening and rolling of metallic materials will introduce non-linear residual stresses. Curing of thermosetting plastics, such as resin impregnated refrasil or fiberglass are also causes of non-linear residual stresses. The residual stresses discussed by the authors in this report are assumed to be biaxial, i.e., in the x and y directions parallel to the surface of the structure. The stresses normal to the surface are assumed to be negligible.

1.2 Method of Approach

The approach used by the authors of this report to determine the residual stresses in a structure is described in Paragraphs 1.2.1, 1.2.2 and 1.2.3 which follow.

1.2.1 Determination of Linear Residual Stresses

The experimental technique used to determine the linear residual stresses utilizes two, two-element, L-type, strain gages in each area where the residual stresses are to be found. One gage is mounted on the top surface and the other gage is mounted on the bottom surface, i.e., the gages are back to back. The gage elements are mounted in the direction of the principal stresses, which are assumed to be known. After mounting the gages and connecting them to a suitable recorder, a zero reading is taken. The gages are then disconnected from the recorder and the element (specimen), which is assumed in this report to be a relatively flat plate of constant thickness, is cut from the structure. The gages are again connected to the recorder and the strains are measured. The released strains will be the readings after the cut minus the readings before the cut. The linear strain which existed in the structural element before the cut will be the released strain with the sign changed. The maximum linear stresses and strains will be on the surface and the stresses may be obtained from the strains by using Equations 2-9 and 2-10 given in Paragraph 2. Equations 2-7 and 2-8 given in Paragraph 2 are for the linear residual stresses at other points between the surfaces for use in the Fortran IV program which is described in Paragraph 5.

1.2.2 Determination of Non-Linear Residual Stresses

After determining the linear residual stresses, the next step is to determine the non-linear residual stresses which still remain in the element. The experimental technique used to determine these stresses is now described. Only one of the two element strain gages previously installed will be used for measuring strains, i.e., the one on the top or the bottom surface. The concept is to measure the change in strains on the top or bottom surface due to removal of layers of material from the opposite surface. Before and after a layer of material is removed, the strain on the opposite surface is measured and also the remaining thickness of the element. Equations 3-35 through 3-44, shown in Paragraph 3, use this data to determine the original non-linear residual stress which existed in each layer before it or any previous layers were removed. A complete development of the equations is given in Paragraph 3.

1.2.3 Fortran IV Residual Stress Program

The Fortran IV Program described in Paragraph 5 solves the equations shown in Paragraphs 2, 3 and 4. The print-out consists of linear residual stresses in the x and y directions from the equations of Paragraph 2, the non-linear residual stresses in the x and y directions from the equations of Paragraph 3 and the total residual stresses from the equations of Paragraph 4. If desired, the output includes a tape for use on the SC-4020 machine to provide an automatic plot of the tabulated data.

1.2.4 Element Size in Relation to Thickness

The tests described in Paragraph 6 were made to determine the minimum length and width of the element (or specimen) in relation to its thickness, which, when cut from a structure, can be used to determine the non-linear residual stresses. It was found that the residual stresses at a particular cross-section begin to drop off when that cross-section is closer to an edge than the thickness of the specimen. It is obvious that the residual stress will become zero at the edge of the specimen. Therefore, to determine the non-linear residual stress by means of an element cut from a structure, the element length and width should be at least twice the element thickness plus the gage length of the strain gage.

1.2.5 Experimental Verification

The tests described in Paragraph 7 were made to check, experimentally, the validity of the equations in Paragraphs 2, 3 and 4 and also the Fortran IV program described in Paragraph 5. The experimental results show excellent agreement with the analytically determined values.

1.3 Typical Uses of the Program

The development work described in this report has simplified and mechanized the determination of the residual stresses to such an extent that only a minimum amount of laboratory work and technician time is required to obtain tabulated and/or graphically displayed data.

The procedure has been used to determine the biaxial residual stress distribution due to curing at various sections along the length of the refrasil liner of a rocket motor nozzle. The refrasil thickness varied from approximately .5" to 1". First a two-element strain gage was installed on the outside and inside surface at various stations along the length of four different nozzles which had had different cure cycles. A lengthwise strip about 3 inches wide, which included all the strain gages, was then cut from each nozzle. Strain gage readings were taken before and after cutting the strip. These readings were recorded on the load sheet for later use in obtaining the linear residual stress due to moment plus axial load. The strip was then cut into blocks about 3 inches long, each block having one strain gage centered on its top and bottom surfaces. The bottom gage was then removed since material would be removed from this side and the gage was no longer needed. The material was removed by an AB Buehler, Ltd. wet belt sander. A total of 21 specimens were cut from four different nozzles. The laboratory time was approximately 2 hours per specimen for installing the strain gages, 1 hour per specimen for the grinding procedure including strain gage readings and thickness measurements, and approximately 3 minutes total computer time to obtain tabular and graphical display of the residual stress distribution for the 21 specimens. Documentation of this work has not been completed or published.

The procedure has also been used to determine the bi-axial residual stress distribution at various sections along the length of .104 inch thick aluminum alloy weld seams. Layers were removed by milling material from one side of the specimens. This work is still in progress.

1.4 Previous Experimental Work

A large amount of experimental work has been performed in the area of residual stress determination. Most of this work can be roughly classified into four separate groups. These consist of material removal procedures, X-ray diffraction techniques, optical, and ultrasonic methods.

1.4.1 Material Removal Procedures

The first of these, the material removal procedure, is by far the most widely used. A characteristic of this technique is the removal of layers from a test specimen by machining, grinding, etching, etc.. Stresses in each layer are then determined from measurement of the dimensional changes experienced by the parent material. A few of the investigators who were instrumental in the development of the theory of the layer removal method were Heyn[3], Sachs[4] and Rembowski[5].

Major variations in the use of this technique usually lie in the manner in which dimensional changes are determined, or material is removed. Dial indicators and acid etching were utilized by Waisman[6], while strain gages and chemical removal were used by Demorest and Leeser[7]. Strain gages were also used by Richards[8], Mack[9], Hanslip[10], Greaves[11] and Davidson[12], while Leaf[13] explored a number of material removal and measurement techniques.

A variation of the layer removal process was suggested by Mathar[14]. Basically, his method consists of drilling a small hole in a test speciman. By this operation, a partial elastic spring back occurs in the immediate vicinity of the hole, and residual stresses at the specimens surface can be determined. This method has been used by Soete[15], Palermo[16] and Riparbelli[17], all of whom used strain gages to determine dimensional changes, and Tokarcik[18] who applied stresscoat to take the required measurements.

The major advantage of the layer removal technique stems from the fact that it is generally easier to apply, and more economical than any of the other methods and it gives a complete picture of the residual stress field. Major disadvantage lies in its destruction of the test specimen. Furthermore, great care must be exercised in its application, so as to minimize introduction of new stresses during the removal operation, and in the measurement of extremely minute dimensional changes.

ADVANTAGES & DIS-ADVANTAGES OF DAC METHOD

The hole drilling method is the least destructive of the material removal techniques, and it permits the evaluation of the residual stresses at what is essentially a point. However, it can generally be used to obtain only those stresses which occur at the test specimen's surface.

1.4.2 X-Ray Diffraction Techniques

Another relatively widely used technique for the determination of residual stresses is that of X-Ray diffraction. The X-Ray method measures spacings between atom planes in a single grain by determining the angular position at which the diffracted X-Ray beam appears on a photographic plate. Changes in spacings between atom planes parallel to the specimens surface and planes inclined at some angle to the surface of the specimen, can be interpreted by changes in angular positions of the corresponding diffracted beams. The stress in any direction on the surface of the specimen can then be related to the change of the diffraction angle of two such measurements by the theory of elasticity. This technique has been used by Barret[19], Stephen[20], Donachie[21], Miller[22] and Harvey[23].

The X-Ray diffraction technique has the considerable advantage of leaving the test specimen intact, but it is quite costly by virtue of the highly specialized equipment required, and can only be used to determine surface stresses at a point.

1.4.3 Optical Techniques

Optical techniques have occasionally been used in the study of residual stresses in certain highly specialized applications. Since glass is a birefringent medium, Riney[24] was able to apply photoelasticity to the determination of residual stresses in electron tubes, and Lettleton[25] used it to study quenching stresses in glass. Nisida[26] utilized the photo-plastic properties of cellulose nitrate to study residual stresses in plastically deformed beams and wedges, and Nye[27] worked with the birefringent properties of silver chloride crystals to investigate crystallographic influences on residual stresses. Lastly, Letner[28] used an interferometric technique in conjuntion with the material removal method to determine residual stresses in rectangular bars.

1.4.4 Ultrasonic Techniques

A relatively recent innovation in the area of experimental residual stress determination is the ultrasonic technique. Firestone[29] has found that residual stresses can cause changes in the velocity and attenuation of ultrasonic waves applied to metal specimens. If the specimen is isotropic, the velocity of the ultrasonic shear wave is independent of the direction of particle motion. If, through the application of stress, this isotropy is

MAY BE OF INTEREST IN APPLICATION TO STRESS WAVE DETECTION SYSTEM destroyed however, the velocity is found to vary with the direction of particle motion. Under these conditions, it was found that a plane polarized ultrasonic shear wave would propagate only if the particle motion was either parallel or perpendicular to the applied stress. This effect was seen to induce birefringence in metallic specimens in much the same way as visible polarized light produces birefringence in certain transparent anisotropic materials.

Rollins[30] has applied this non-destructive technique to the study of residual stresses in aluminum and steel specimens.

1.5 Suggested Additional Work

One of the limitations of the procedure outlined in this report is that the cross-section, in which the non-linear residual stress distribution is desired, be rectangular. By making some revisions in the equations, so that the area and moment of inertia of irregular cross-sections could be continuously determined, the Fortran IV program could be revised to handle any shape or cross-section.

Another limitation of the procedure outlined in the report is the assumption that any layer of material parallel to the top and bottom surface of the specimen is the same as every other layer. In sandwich type construction this is not true. By proper revision of some of the equations, and the inclusion of these revisions in the Fortran IV program, it would also be possible to eliminate this limitation.

Good

2. EXPERIMENTAL DETERMINATION OF LINEAR RESIDUAL STRESSES IN AN ORTHOTROPIC MATERIAL DUE TO THE RELEASE OF MOMENT AND AXIAL LOAD

The experimental procedure for determining the linear residual stresses and the assumptions made are outlined in paragraph 1.2.1. The purpose of this section is to show the development of the equations for maximum stress at the surfaces and the average stress which exists in any arbitrary layer chosen between the surfaces. An equation for the average stress in any layer is necessary so that this stress may be added to the non-linear stress which is determined by the procedure outlined in Paragraph 3.

2.1 Analysis Development

The strain distribution in the x-direction which existed in the element before it was cut from the structure is assumed to be linear as shown in Figure 2-1. This assumption is justified by the fact that a stress-strain curve obtained from unloading a tensile coupon is a straight line. This is true even if the coupon had deformed plastically during loading. A linear strain distribution is also assumed to exist in the y-direction. The strain distributions are due to a moment and axial load in both the x and y directions. From these strains, which are released when the element is cut out of the structure, it is possible to find the stress distribution.

STRAIN DISTRIBUTION BEFORE ELEMENT IS CUT FROM PART
AND IDENTIFICATION OF ELEMENT LAYERS

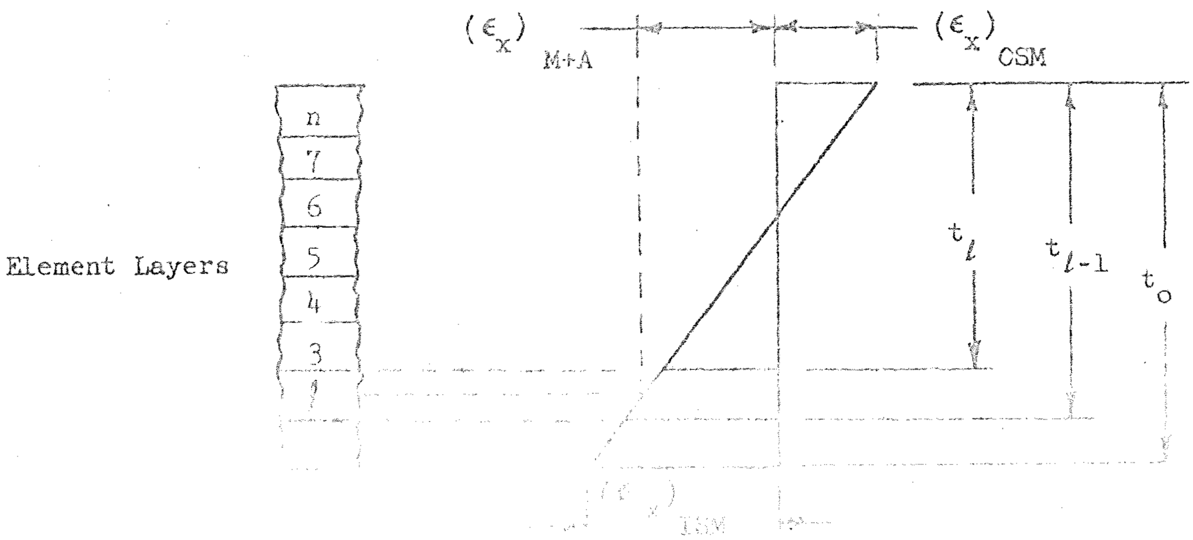


FIGURE 2-1

Define the following terms as:

$(\epsilon_x)_{OSM_o}$ = strain reading along the x-axis on the outside surface due to moment plus axial load before the element is cut from the structure. Positive if indicating a unit length increase of the gage. (in./in.)

$(\epsilon_x)_{OSM}$ = strain which existed on the outside surface of the element before cutting the element from the structure. (in./in.)

Then using the following subscripts,

- x, y = along the x and y-axes respectively.
- OSM = on the outside surface due to moment plus axial load.
- ISM = on the inside surface due to moment plus axial load.
- o, a = before and after the cut respectively.
- ℓ = any layer
- n = last layer

It follows that,

$$(\epsilon_x)_{OSM} = (\epsilon_x)_{OSM_o} - (\epsilon_x)_{OSM_a} \qquad (2\text{-}1)$$

$$(\epsilon_y)_{OSM} = (\epsilon_y)_{OSM_o} - (\epsilon_y)_{OSM_a} \qquad (2\text{-}2)$$

$$(\epsilon_x)_{ISM} = (\epsilon_x)_{ISM_o} - (\epsilon_x)_{ISM_a} \qquad (2\text{-}3)$$

$$(\epsilon_y)_{ISM} = (\epsilon_y)_{ISM_o} - (\epsilon_y)_{ISM_a} \qquad (2\text{-}4)$$

Now let

$(\epsilon_x)_{\ell_{M+A}}$ = strain which existed at the mid-point of the ℓth layer before cutting the element from the structure due to moment plus axial load (in./in.)

From inspection of Figure 1,

$$\frac{(\epsilon_x)_{OSM} - (\epsilon_x)_{ISM}}{t_o} = \frac{(\epsilon_x)_{OSM} - (\epsilon_x)_{\ell_{M+A}}}{\frac{t_\ell - t_{\ell-1}}{2}}$$

+?

$$(\epsilon_x)_{\ell_{M+A}} = (\epsilon_x)_{OSM} - \frac{\left[(\epsilon_x)_{OSM} - (\epsilon_x)_{ISM} \right] (t_\ell + t_{\ell-1})}{2\, t_o} \qquad (2\text{-}5)$$

Similarly

$$(\epsilon_y)_{\ell_{M+A}} = (\epsilon_y)_{OSM} - \frac{\left[(\epsilon_y)_{OSM} - (\epsilon_y)_{ISM} \right] (t_\ell + t_{\ell-1})}{2\, t_o} \qquad (2\text{-}6)$$

To obtain the residual stress in any layer due to the release of moment plus axial load, Equation 9 given on page 3 of Reference (1) may be used.

$$(S_x)_{\ell_{M+A}} = \frac{E_x}{1 - \mu_{yx}\,\mu_{xy}} \left[(\epsilon_x)_{\ell_{M+A}} + \mu_{xy} (\epsilon_y)_{\ell_{M+A}} \right] \qquad (2\text{-}7)$$

$$(S_y)_{\ell_{M+A}} = \frac{E_y}{1 - \mu_{yx}\,\mu_{xy}} \left[(\epsilon_y)_{\ell_{M+A}} + \mu_{yx} (\epsilon_x)_{\ell_{M+A}} \right] \qquad (2\text{-}8)$$

For the last layer, n, Equation 2-7 and 2-8 may be used merely by replacing ℓ with n.

The outside surface stresses may be obtained from Equations 2-9 and 2-10.

$$(S_x)_{OSM} = \frac{E_x}{1 - \mu_{yx}\,\mu_{xy}} \left[(\epsilon_x)_{OSM} + \mu_{xy} (\epsilon_y)_{OSM} \right] \qquad (2\text{-}9)$$

$$(S_y)_{OSM} = \frac{E_y}{1 - \mu_{yx}\,\mu_{xy}} \left[(\epsilon_y)_{OSM} + \mu_{yx} (\epsilon_x)_{OSM} \right] \qquad (2\text{-}10)$$

The inside surface stresses are found by using subscript ISM instead of OSM in Equations 2-9 and 2-10.

3. EXPERIMENTAL DETERMINATION OF THE NON-LINEAR RESIDUAL STRESSES IN AN ORTHOTROPIC MATERIAL BY THE USE OF STRAIN GAGES AND MATERIAL REMOVAL

This paragraph shows all the concepts involved and derives the necessary equations to determine the non-linear residual stress distribution in an orthotropic material by the use of strain gages and material removal. If the material is isotropic the method is equally applicable merely by using the same modulus of elasticity and Poisson's ratio in both the x and y directions. The element used, shown in Figure 3-1, is assumed to have been removed from a structure in which the non-linear residual stress distribution is to be determined.

DESCRIPTION OF ELEMENT

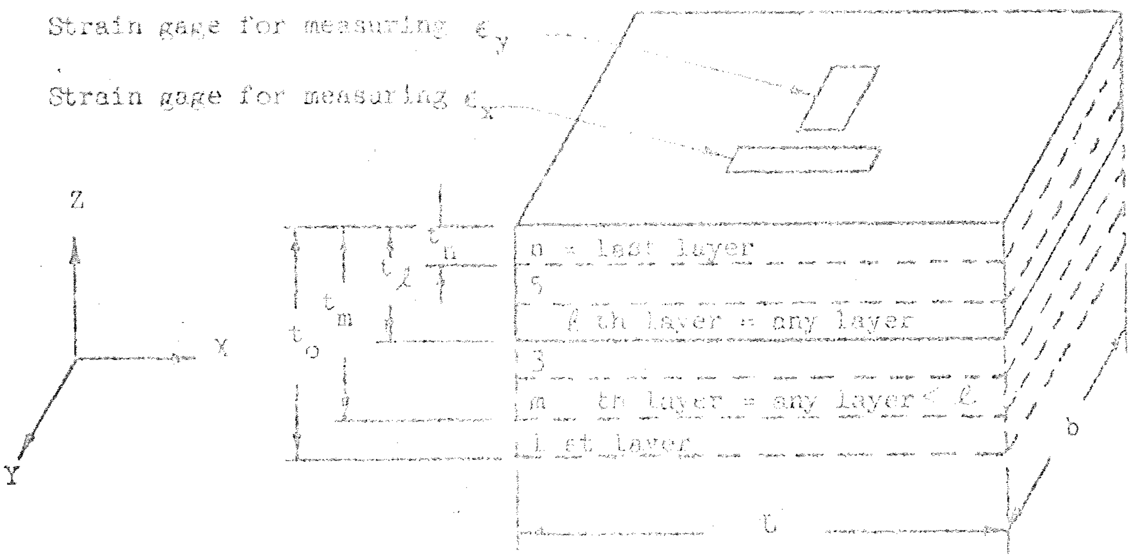


FIGURE 3-1

NOTE: L and b need not be equal but each should be equal to or greater than 2 t_c plus the gage length of the strain gage, see Paragraph 6.

3.1 Symbols

$(\epsilon_x)_{\ell-1}$ = strain reading along the x-axis before removal of the ℓth layer of material. Positive if indicating a unit length increase of the gage. (in./in.)

$(\epsilon_x)_{\ell}$ = strain reading along the x-axis after removal of the ℓth layer of material. Positive if indicating a unit length increase of the gage. (in./in.)

$(\Delta\epsilon_x)_{\ell}$ = change in strain on the top surface along the x-axis due to removal of the ℓth layer from the bottom surface. Positive if unit length increases. (in./in.)

$(S_x)_{\ell}$ = average stress along the x-axis which existed in the ℓth layer before it was removed but after ℓ-1 layers were removed. Positive if tension. (psi)

$(\Delta S_x)_{\ell,m}$ = average change in stress along the x-axis in the ℓth layer due to the removal of the mth layer. $m<\ell$. Positive if tension. (psi)

$(S_x)_{n_T}$ = total non-linear residual stress in the nth layer, i.e., last layer. (psi)

$(S_x)_{\ell_T}$ = total non-linear residual stress in the ℓth layer, i.e., any layer. (psi)

μ_{yx} = Poisson's ratio in the y direction due to a load in the x direction.

μ_{xy} = Poisson's ratio in the x direction due to a load in the y direction.

$(\Delta M_x)_{\ell}$ = moment in the xz plane on the cross-section of thickness t_{ℓ} required to cause the same change in strain as was caused by the removal of the moment due to the removal of the ℓth layer of material. Compression in top is positive. (in.lb.)

$(\Delta P_x)_\ell$ = axial load along the x-axis on the cross-section of thickness t_ℓ required to cause the same change in strain as was caused by the removal of the axial load due to the removal of the ℓth layer of material. Tension in top is positive. (lb.)

$z_{\ell,m}$ = distance from the center of the ℓth layer to the neutral axis of the section after removal of the mth layer. Positive if the center of the ℓth layer is below the neutral axis of the section. (in.)

E_x = modulus of elasticity in the x-direction. (psi)

Subscripts:

n = last layer or total number of layers

ℓ = any layer

m = any layers less than ℓ which have been removed, see Figure 3-7.

x, y = along the x and y-axis respectively, see Figure 3-1

T = total

Other notation is defined where used.

3.2 Spring Beam Analogy of Layer Removal Process

The concepts involved in deriving the basic relationships between stress and strain, needed for determining the non-linear residual stresses in a beam by the layer removal process, may be most clearly demonstrated by idealizing the beam into a series of springs as shown in Figure 3-3. Each spring represents a layer of material which has a constant residual stress across it.

A discussion and analysis of an idealized spring-beam will therefore be made prior to the development of a more general analysis for the actual beam. In the analysis of the idealized spring-beam, stresses and strains in one direction only will be considered and, therefore, Poisson's ratio may be neglected. The eff... of stresses and strains in two directions, including Poisson's ratio, will be considered in the more general analysis of Paragraph 3.3.

3.2.1 Change in Moment and Axial Load Due to Removal of Layer

The value of $(\Delta M_x)_1$, $(\Delta P_x)_1$, shown in Figure 3-2 are the moments and axial loads which when applied to the beam of thickness t_1 will cause the same change in strain readings on the top surface as those caused by the removal of the 1st layer.

MOMENTS AND AXIAL LOADS CAUSED BY REMOVAL OF A LAYER

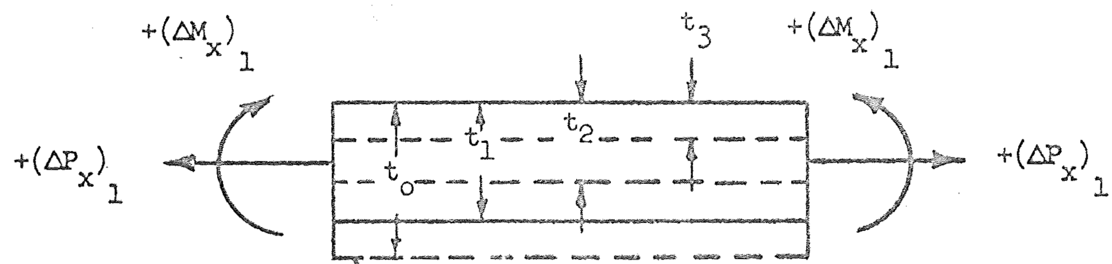


FIGURE 3-2

The concept may best be illustrated by an example of a simulated beam composed of four springs as shown in Figure 3-3

SIMULATED BEAM BEFORE CUT

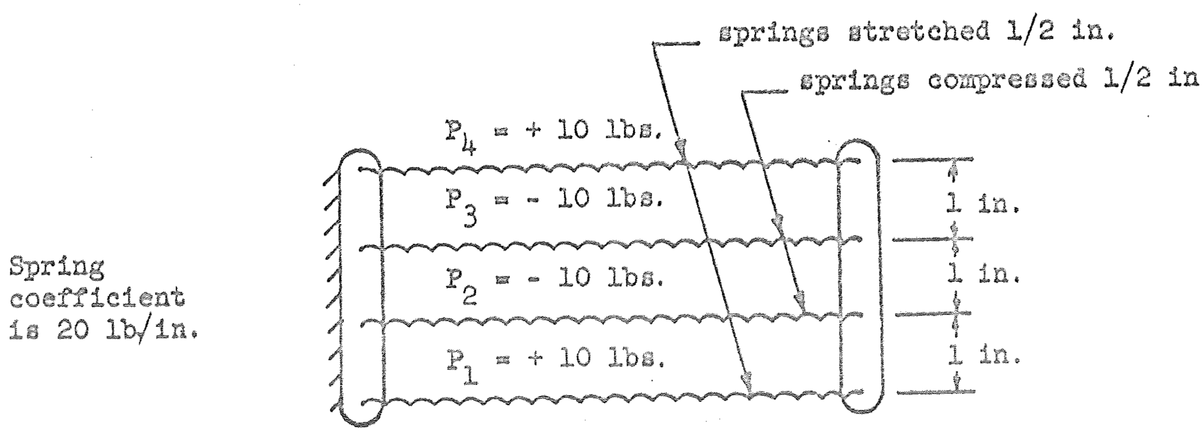


FIGURE 3-3

SIMULATED BEAM AFTER CUT

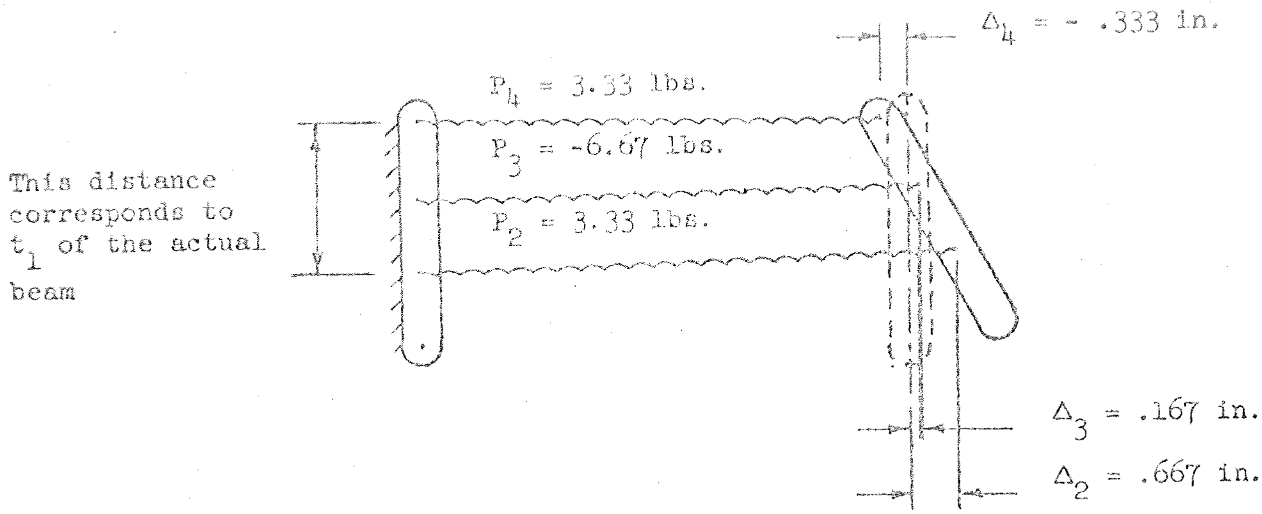


FIGURE 3-4

$P_4 = 10 + \Delta_4 (20)$

$P_3 = -10 + \Delta_3 (20)$

$P_2 = -10 + \Delta_2 (20)$

$F = 0$

$P_2 + P_3 + P_4 = 0$

$M = 0$

$P_4 (1) = P_2 (1)$

Also

$(\Delta_3 - \Delta_4 = \Delta_2 - \Delta_3)$

Simultaneous solution of these equations results in the loads and deflections of the springs listed in Figure 3-4

It is noted that the removal of spring No. 4 on the simulated beam of Figure 3-3 caused a change in deflection of the remaining springs of Figure 3-4 of

$\Delta_4 = -.333$ in.

$\Delta_3 = .167$ in.

$\Delta_2 = .667$ in.

Aren't there an infinite number of combinations of axial load and bending moments which may be applied to the "unbalanced" "beam" of fig. 3-4 to restore the 10 lbs in spring #4? If this is true, and since we can only determine the change in strain resulting from removing a layer, how do we determine whether the residual force is bending, axial load, or both?

It is obvious that if a moment of 10 x 2 = 20 in.lb. and an axial load of 10 lb ←, which is the effect of spring No. 4, is applied to the beam of Figure 3-4, it will bring the elements of this beam back to their original load and deflection of Figure 3-3, thus

$$P_4 = 3.33 + \frac{20}{2} - \frac{10}{3} = 10 \text{ lbs.}$$

$$P_3 = -6.67 - \frac{10}{3} = -10 \text{ lbs.}$$

$$P_2 = 3.33 + \frac{20}{2} - \frac{10}{3} = 10 \text{ lbs.}$$

The result is as shown in Figure 3-5.

RESTORING AXIAL LOAD AND MOMENT

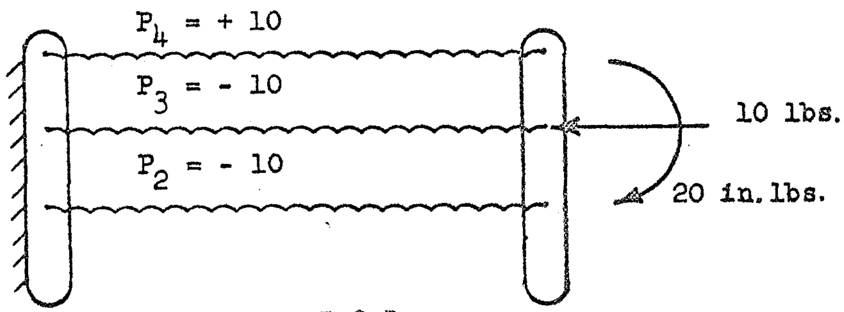


FIGURE 3-5

Now apply a moment of 20 in.lb. and an axial load of 10 lb. → to the simulated beam of Figure 3-5 and the result will be the same as the beam shown in Figure 3-4 which has no external moment or axial load applied to it. Therefore, it has been demonstrated that the application of a moment of 20 in.lb. and axial load of 10 lb. →, which corresponds to $(\Delta M_x)_1$ and $(\Delta P_x)_1$, to the weakened beam of thickness t_1 will cause the same change in strain readings on the top surface as those caused by the removal of the bottom spring which corresponds to the 1st layer of the actual beam.

3.2.2 Steps Involved in a Simple Spring-Beam Analogy

Having determined the loads and deflections in the springs of an idealized spring-beam and the direction of $(\Delta M_x)_1$, and $(\Delta P_x)_1$ required to cause the same change in load on the top spring (or surface) as that caused by the removal of the bottom spring (or layer), it is now desirable to correlate residual stresses in an actual beam to the loads in the springs of the idealized spring-beam. A study of the following steps will add clarity to the procedure and analysis development of Paragraph 3.3.

Step 1:

Cut spring 1. Measure the strain in the top spring 4. If the springs shown in Figure 3-3 are assumed to be 1 in. long then the strain in the top spring 4 is -.333 in. In an actual beam in which the residual stress is to be determined this value would be the quantity obtained by the strain gage reading after removing the first strip minus the reading before removing the first strip. Each spring will be considered to represent a 1 sq. in. area then the modulus of elasticity = E = 10/.5 = 20 psi.

Step 2:

The measured change in strain after cutting spring 1 is the result of a moment $(\Delta M_x)_1$ and an axial load, $(\Delta P_x)_1$, which, because of the removal of spring 1, have been applied to the remaining three springs. Positive $(\Delta M_x)_1$ is and positive $(\Delta P_x)_1$ is .

Step 3:

The determination of $(\Delta M_x)_1$ and $(\Delta P_x)_1$ is as follows:

$(\Delta S_x)_{4,1}$ = change in stress in spring 4 due to the removal of spring 1.

$$(\Delta S_x)_{4,1} = (\Delta \epsilon_x)_{4,1}\, E = -.333\ (20) = -6.66 \text{ psi}$$

$y_{4,1}$ = distance from the center of the 4th spring to the neutral axis of the section after removal of the 1st spring. Positive if the center of the 4th spring is below the neutral axis of the section.

$y_{4,1}$ = -1 Reference Figure 3-4

$$I_1 = \Sigma A_y{}^2 = 2\ (1)\ (-1)^2 = 2 \text{ in.}^4$$

$$(\Delta S_x)_{4,1} = \frac{(\Delta M_x)_1\ y_{4,1}}{I_1} + \frac{(\Delta P_x)_1}{A_1}$$

$$(\Delta M_x)_1 = (\Delta P_x)_1\ (2)$$

$$(\Delta S_x)_{4,1} = \frac{2\ (\Delta P_x)_1\ (-1)}{2} + \frac{(\Delta P_x)_1}{3}$$

$$-6.66 = -(\Delta P_x)_1 + .333\ \Delta P_x$$

$$(\Delta P_x)_1 = 10 \text{ lb} \longrightarrow$$

and $(\Delta M_x)_1 = (10)\ (2) = 20$ in. lb.

Step 4:

From the value of $(\Delta M_x)_1$ found above it is possible to determine the stress $(S_x)_1$ which existed in spring 1 before the cut, thus

$$(S_x)_1\ (1)\ (2) = (\Delta M_x)_1 = 20 \text{ in. lb.}$$

$$(S_x)_1 = 10 \text{ psi}$$

This agrees with the original model of Figure 3-3 and completes the analysis of spring 1.

Step 5:

Now it is necessary to determine the change in stresses in springs 2, 3 and 4 due to $(\Delta M_x)_1$ and $(\Delta P_x)_1$.

$$(\Delta S_x)_{2,1} = \frac{(\Delta M_x)_1\ y_{2,1}}{I} + \frac{(\Delta P_x)_1}{A}$$

$$= \frac{20\ (1)}{2} + \frac{10}{3} = 10 + 3.333 = 13.333 \text{ psi}$$

$$(\Delta S_x)_{3,1} = \frac{20\ (0)}{2} + \frac{10}{3} = 0 + 3.333 = 3.333 \text{ psi}$$

$$(\Delta S_x)_{4,1} = \frac{20\ (-1)}{2} + \frac{10}{3} = -10 + 3.333 = -6.667 \text{ psi}$$

These changes agree with the changes which occurred between the model of Figures 3-3 and 3-4.

Step 6:

So far the original stress in spring 1 has been determined in Step 1 through 4. The change in stress in springs 2, 3 and 4 have been determined in Step 5. Now, in order to determine the original stress in spring 2, it is necessary to know the stress which exists in spring 2 after removing spring 1. This is done by removing spring 2, noting the change in strain in spring 4, and then repeating Steps 1, 2, 3 and 4 as was done previously to determine the original stress existing in spring 1.

The additional strain in spring 4 due to the removal of spring 2 is, by inspection of Figure 3-4, equal to -3.33/20 = -.1667 inches. In an actual beam in which the residual stress is being determined this value would be the quantity obtained by the strain gage reading after the removal of the second strip minus the reading before removing the second strip. Then

$$(\Delta\epsilon_x)_{4,2} = -.1667 \text{ in.}$$

$$(\Delta S_x)_{4,2} = (\Delta\epsilon_x)_{4,2}\ E = -.1667\ (20) = -3.33 \text{ psi}$$

$$y_{4,2} = -.5 \text{ in.}$$

$$I_2 = \Sigma A_y{}^2 = 2\ (1)\ (.5)^2 = .5 \text{ in.}^4$$

$$(\Delta S_x)_{4,2} = \frac{(\Delta M_x)_2\ y_{4,2}}{I} + \frac{(\Delta P_x)_2}{A}$$

$$(\Delta M_x)_2 = (\Delta P_x)_2\ (1.5)$$

$$(\Delta S_x)_{4,2} = \frac{(\Delta P_x)_2\ (1.5)\ (-.5)}{.5} + \frac{(\Delta P_x)_2}{2}$$

$$-3.33 = -1.5\ (\Delta P_x)_2 + .5\ (\Delta P_x)_2$$

$(\Delta P_x)_2 \quad = 3.33 \text{ lb} \longrightarrow$

and

$(\Delta M_x)_2 \quad = (3.33)\ (1.5) = 5 \text{ in. lb.}$

$(S_x)_2\ (1)\ (1.5) = (\Delta M_x)_2 = 5 \text{ in. lb.}$

$(S_x)_2 \quad = 3.33 \text{ psi}$

This agrees with the model of Figure 3-4 and the value $(S_x)_2$ is the value of the stress in spring 2 determined from the change in strain in spring 4 which occurred due to the cutting of spring 2.

Step 7:

In order to determine the total original stress in spring 2, $(S_x)_{2_T}$, it is necessary to <u>subtract</u> the change in stress which occurred in spring 2 due to the cutting of spring 1, $(\Delta S_x)_{2,1}$, from the stress, $(S_x)_2$, thus

$$(S_x)_{2_T} = (S_x)_2 - (\Delta S_x)_{2,1}$$

$$= 3.33 - 13.333 = -10.00 \text{ psi}$$

This agrees with the original model of Figure 3-3 and completes the analysis of spring 2.

Step 8:

Determine the change in stress in spring 3 and 4 from the cutting of spring 2. This is similar to Step 5.

$$(\Delta S_x)_{3,2} = \frac{(\Delta M_x)_2\ y_{3,2}}{I_2} + \frac{(\Delta P_x)_2}{A_2}$$

$$= \frac{5\ (.5)}{.5} + \frac{3.33}{2} = 6.67 \text{ psi}$$

$$(\Delta S_x)_{4,2} = \frac{5\ (-.5)}{.5} + \frac{3.33}{2} = -3.33 \text{ psi}$$

These changes would cause the stresses in springs 3 and 4 to go to zero, which, by inspection of Figure 3-4, is what would actually happen.

Step 9:

Find $(S_x)_3$ by cutting spring 3. It is obvious from step 7 that after cutting spring 2 the stresses in springs 3 and 4 will go to zero. Therefore, to cut spring 3 would cause no additional strain in spring 4.

Then $(\Delta \epsilon_x)_{4,3} = 0$. In an actual beam in which the residual stress is being determined this value would be the quantity obtained by the strain gage reading after the removal of the third strip minus the reading before removing the third strip.

$$(\Delta S_x)_{4,3} = (\Delta \epsilon_x)_{4,2} \; E = 0$$

$$(\Delta P_x)_3 = 0$$

$$(\Delta M_x)_3 = 0$$

$$(S_x)_3 = 0$$

Step 10:

Similar to Step 7, the total original stress in spring 3, $(S_x)_{3_T}$, is obtained as follows:

$$(S_x)_{3_T} = (S_x)_3 - (\Delta S_x)_{3,1} - (\Delta S_x)_{3,2}$$

$$= 0 - 3.333 - 6.67 = -10 \text{ psi}$$

This agrees with the original model of Figure 3-3 and completes the analysis of the idealized spring beam.

3.3 Procedure and Analysis Development

The following paragraphs describe the procedure and present the development of the final equations used to determine the non-linear residual stress distribution in a structural element by the use of two strain gages, one to measure x-direction strains and one to measure y-direction strains, in conjunction with the removal of material. If it is more convenient a single strain gage consisting of two elements at 90° to each other may be used.

3.3.1 Determination Of The Stress in the ℓth Layer Due to the Removal of the ℓth Layer

Place the strain gages on the top surface. Read $(\epsilon_x)_o$, $(\epsilon_y)_o$ and measure t_o. Then remove layer No. 1 from the bottom surface. Read $(\epsilon_x)_1$, $(\epsilon_y)_1$ and measure t_1.

$$(\Delta\epsilon_x)_1 = (\epsilon_x)_1 - (\epsilon_x)_o \tag{3-1}$$

$$(\Delta\epsilon_y)_1 = (\epsilon_y)_1 - (\epsilon_y)_o \tag{3-2}$$

Or in general terms

$$(\Delta\epsilon_x)_\ell = (\epsilon_x)_\ell - (\epsilon_x)_{\ell-1} \tag{3-3}$$

$$(\Delta\epsilon_y)_\ell = (\epsilon_y)_\ell - (\epsilon_y)_{\ell-1} \tag{3-4}$$

Assume that $(\Delta\epsilon_x)_1$ and $(\Delta\epsilon_y)_1$ were caused by an average stress $(S_x)_1$ and $(S_y)_1$ which existed in the first layer before its removal. Then the change in moment in the xz plane on the cross-section of thickness t_1 is

$$(\Delta M_x)_1 = (S_x)_1 \, (t_o - t_1) \, (b) \, \left(\frac{t_o}{2}\right) \tag{3-5}$$

This causes compression in the top surface if $(S_x)_1$ is tension.

Note the dimension $\frac{t_o}{2}$ shown in Figure 3-6 is $\frac{t_o + t_1}{2} - \frac{t_1}{2} = \frac{t_o}{2}$

DETERMINATION OF NEUTRAL AXIS

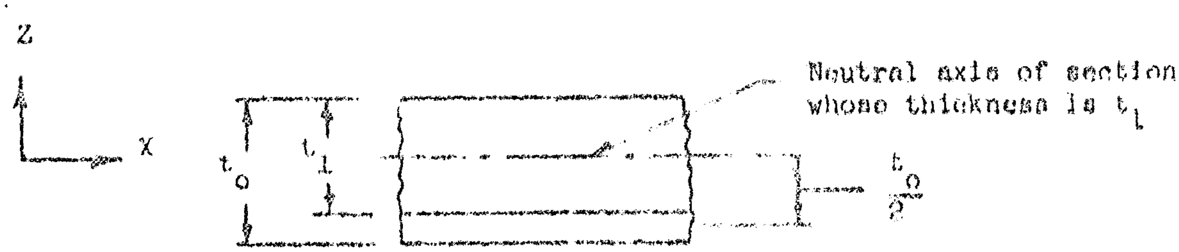


FIGURE 3-6

Similarly in the yz direction,

$$(\Delta M_y)_1 = (S_y)_1 \; (t_o - t_1) \; (L) \; (\frac{t_o}{2}) \tag{3-6}$$

This causes compression in the top surface if $(S_y)_1$ is tension.

The change in the axial load along the x-axis on the cross section of thickness t_1 is

$$(\Delta P_x)_1 = (S_x)_1 \; (t_o - t_1) \; (b) \tag{3-7}$$

This causes tension in the top surface if $(S_x)_1$ is tension.

Similarly along the y-axis

$$(\Delta P_y)_1 = (S_y)_1 \; (t_o - t_1) \; (L) \tag{3-8}$$

This causes tension in the top surface if $(S_y)_1$ is tension.

Then using Equation 6 given on Page 3 of Reference 1

$$(\Delta \epsilon_x)_1 = - \frac{(\Delta M_x)_1 \frac{t_1}{2}}{E_x \; (\frac{1}{12}) \; (b) \; (t_1{}^3)} + \mu_{xy} \frac{(\Delta M_y)_1 \frac{t_1}{2}}{E_y \; (\frac{1}{12}) \; (L) \; (t_1{}^3)} + \frac{(\Delta P_x)_1}{E_x \; b \; t_1}$$

$$- \mu_{xy} \frac{(\Delta P_y)_1}{E_y \; L \; t_1} = - \frac{6(\Delta M_x)_1}{E_x \; b \; t_1{}^2} + \mu_{xy} \frac{6(\Delta M_y)_1}{E_y \; L \; t_1{}^2} + \frac{(\Delta P_x)_1}{E_x \; b \; t_1} - \mu_{xy} \frac{(\Delta P_y)_1}{E_y \; L \; t_1} \tag{3-9}$$

Similarly

$$(\Delta \epsilon_y)_1 = - \frac{6(\Delta M_y)_1}{E_y \; L \; t_1{}^2} + \mu_{yx} \frac{6(\Delta M_x)_1}{E_x \; b \; t_1{}^2} + \frac{(\Delta P_y)_1}{E_y \; L \; t_1} - \mu_{yx} \frac{(\Delta P_x)_1}{E_x \; b \; t_1} \tag{3-10}$$

Substitute Equations 3-5, 3-6, 3-7 and 3-8 into Equation 3-9

$$(\Delta \epsilon_x)_1 = - \frac{6(S_x)_1 \ (t_o - t_1) \ (\frac{t_o}{2})}{E_x \ t_1^{\ 2}} + \mu_{xy} \frac{6(S_y)_1 \ (t_o - t_1) \ (\frac{t_o}{2})}{E_y \ t_1^{\ 2}}$$

$$+ \frac{(S_x)_1 \ (t_o - t_1)}{E_x \ t_1} - \mu_{xy} \frac{(S_y)_1 \ (t_o - t_1)}{E_y \ t_1}$$

$$= \frac{(t_o - t_1)}{t_1} \left[- \frac{3 \ (S_x)_1 \ t_o}{E_x \ t_1} + \mu_{xy} \frac{3(S_y)_1 \ t_o}{E_y \ t_1} + \frac{(S_x)_1}{E_x} - \mu_{xy} \frac{(S_y)_1}{E_y} \right] \tag{3-11}$$

Similarly

$$(\Delta \epsilon_y)_1 = \frac{t_o - t_1}{t_1} \left[- \frac{3(S_y)_1 \ t_o}{E_y \ t_1} + \mu_{yx} \frac{3(S_x)_1 \ t_o}{E_x \ t_1} + \frac{(S_y)_1}{E_y} - \mu_{yx} \frac{(S_x)_1}{E_x} \right] \tag{3-12}$$

Solve Equation 3-11 and Equation 3-12 simultaneously by multiplying Equation 3-12 by μ_{xy} and add the result to Equation 3-11.

$$(\Delta \epsilon_x)_1 + \mu_{xy} \ (\Delta \epsilon_y)_1$$

$$= \frac{(t_o - t_1)}{t_1} \left[- \frac{3(S_x)_1 \ t_o}{E_x \ t_1} + \mu_{xy} \mu_{yx} \frac{3(S_x)_1 t_o}{E_x \ t_1} + \frac{(S_x)_1}{E_x} - \mu_{xy} \mu_{yx} \frac{(S_x)_1}{E_x} \right]$$

$$= \frac{(t_o - t_1)}{t_1 \ E_x} \left[- \frac{3 \ t_o}{t_1} (1 - \mu_{xy} \mu_{yx}) + (1 - \mu_{xy} \ \mu_{yx}) \right] (S_x)_1$$

$$= (\frac{t_o - t_1}{t_1 \ E_x}) \ (1 - \mu_{xy} \ \mu_{yx}) \ (- \frac{3 \ t_o}{t_1} + 1) \ (S_x)_1$$

$$= (\frac{t_o - t_1}{t_1^{\ 2} \ E_x}) \ (1 - \mu_{xy} \ \mu_{yx}) \ (- 3 \ t_o + t_1) \ (S_x)_1 \tag{3-13}$$

$$(S_x)_1 = - \frac{t_1^2 E_x \left[(\Delta \epsilon_x)_1 + \mu_{xy} (\Delta \epsilon_y)_1 \right]}{(t_o - t_1)(3 t_o - t_1)(1 - \mu_{xy} \mu_{yx})} \tag{3-14}$$

And similarly

$$(S_y)_1 = - \frac{t_1^2 E_y \left[(\Delta \epsilon_y)_1 + \mu_{yx} (\Delta \epsilon_x)_1 \right]}{(t_o - t_1)(3 t_o - t_1)(1 - \mu_{xy} \mu_{yx})} \tag{3-15}$$

The value of $(S_x)_\ell$ and $(S_y)_\ell$ may be written immediately by its similarity to $(S_x)_1$ and $(S_y)_1$ which are given by Equations 3-14 and 3-15:

$$(S_x)_\ell = - \frac{t_\ell^2 E_x \left[(\Delta \epsilon_x)_\ell + \mu_{xy} (\Delta \epsilon_y)_\ell \right]}{(t_{\ell-1} - t_\ell)(3 t_{\ell-1} - t_\ell)(1 - \mu_{xy} \mu_{yx})} \tag{3-16}$$

$$(S_y)_\ell = - \frac{t_\ell^2 E_y \left[(\Delta \epsilon_y)_\ell + \mu_{yx} (\Delta \epsilon_x)_\ell \right]}{(t_{\ell-1} - t_\ell)(3 t_{\ell-1} - t_\ell)(1 - \mu_{xy} \mu_{yx})} \tag{3-17}$$

Let

$$\frac{E_x}{1 - \mu_{xy} \mu_{yx}} = M \text{ and } \frac{E_y}{1 - \mu_{xy} \mu_{yx}} = N$$

Then

$$(S_x)_\ell = - \frac{t_\ell^2 M \left[(\Delta \epsilon_x)_\ell + \mu_{xy} (\Delta \epsilon_y)_\ell \right]}{(t_{\ell-1} - t_\ell)(3 t_{\ell-1} - t_\ell)} \tag{3-18}$$

$$(S_y)_\ell = - \frac{t_\ell^2 N \left[(\Delta \epsilon_y)_\ell + \mu_{yx} (\Delta \epsilon_x)_\ell \right]}{(t_{\ell-1} - t_\ell)(3 t_{\ell-1} - t_\ell)} \tag{3-19}$$

Note that after the removal of each layer it is necessary to read $(\epsilon_x)_\ell$ and $(\epsilon_y)_\ell$ so that these values may be used in Equations 3-3 and 3-4 to determine $(\Delta \epsilon_x)_\ell$ and $(\Delta \epsilon_y)_\ell$. Also it is necessary to measure t_ℓ.

3.3.2 Determination of Stress in the ℓth Layer Due to the Removal of the 1st Layer

$(S_x)_1$ and $(S_y)_1$ determined by Equation 3-14 and 3-15 are the average stresses which existed in layer 1 before it was removed, but the stresses were found by removing the layer. The removal of this layer caused a change in moment of $(\Delta M_x)_1$ and $(\Delta M_y)_1$ and a change in axial load of $(\Delta P_x)_1$ and $(\Delta P_y)_1$ on the section whose thickness is t_1. These changes in moment and axial load caused changes in the stresses in the next layers which are not yet removed. $(\Delta M_x)_1$, $(\Delta M_y)_1$, $(\Delta P_x)_1$, and $(\Delta P_y)_1$ can be determined by Equations 3-5, 3-6, 3-7, and 3-8. The change in stress in the 2nd, 3rd, and ℓth layers caused by the removal of the first layer is found as follows:

Let

$(\Delta S_x)_{2,1}$ = change in stress in the 2nd layer due to the removal of the 1st layer.

$(\Delta S_x)_{3,1}$ = change in stress in the 3rd layer due to the removal of the 1st layer.

$(\Delta S_x)_{\ell,1}$ = change in stress in the ℓth layer due to the removal of the 1st layer.

$z_{\ell,1}$ = distance from center of the ℓth layer to neutral axis of the section after removal of the 1st layer. Positive if center of ℓth layer is below neutral axis of section.

DETERMINATION OF $z_{\ell,m}$

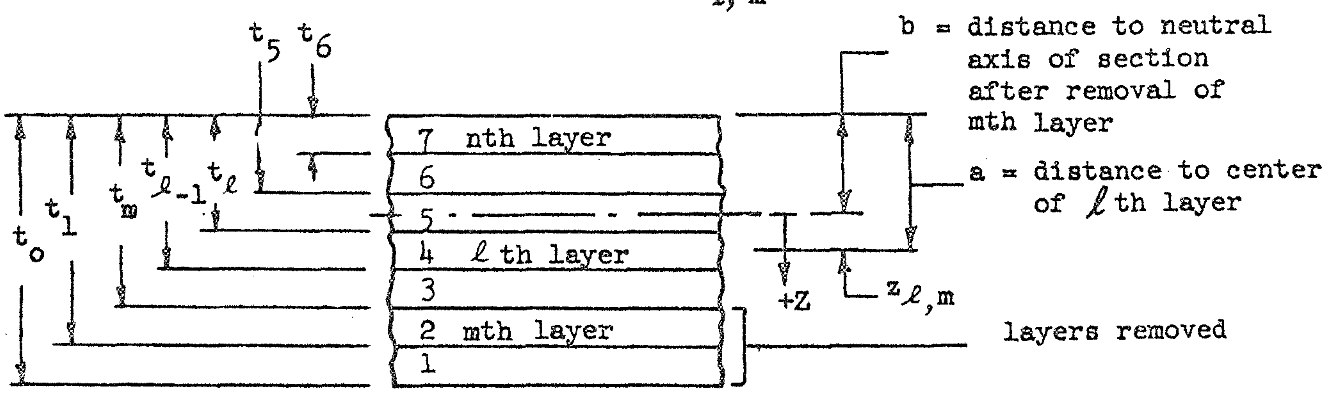


FIGURE 3-7

$$\tau_{\ell,\,m} = a - b - \frac{t_{\ell-1} + t_\ell}{2} - \frac{t_m}{2}$$

$$= \frac{1}{2}\,(t_{\ell-1} + t_\ell - t_m) \tag{3-20}$$

By inspection of Figure 3-2

$$(\Delta S_x)_{2,1} = \frac{(\Delta M_x)_1\; z_{2,1}}{I_1} + \frac{(\Delta P_x)_1}{A_1}$$

$$= \frac{(\Delta M_x)_1\; z_{2,1}}{(\frac{1}{12})\,(b)\,t_1^{\,3}} + \frac{(\Delta P_x)_1}{b\;t_1}$$

$$= \frac{12\,(S_x)_1\,(t_o - t_1)\,(b)\,(\frac{t_o}{2})\,(z_{2,1})}{b\;t_1^{\,3}} + \frac{(S_x)_1\,(t_o - t_1)\,(b)}{b\;t_1}$$

$$= \frac{6\,(S_x)_1\,(t_o - t_1)\,t_o\,z_{2,1}}{t_1^{\,3}} + \frac{(S_x)_1\,(t_o - t_1)}{t_1} \tag{3-21}$$

Now let

$$B_1 = \frac{6\,(t_o - t_1)\,t_o}{t_1^{\,3}} \quad \text{and} \quad C_1 = \frac{t_o - t_1}{t_1}$$

Then

$$(\Delta S_x)_{2,1} = B_1\,z_{2,1}\,(S_x)_1 + C_1\,(S_x)_1 = (B_1\,z_{2,1} + C_1)\,(S_x)_1 \tag{3-22}$$

$$(\Delta S_x)_{3,1} = B_1\,z_{3,1}\,(S_x)_1 + C_1\,(S_x)_1 = (B_1\,z_{3,1} + C_1)\,(S_x)_1 \tag{3-23}$$

$$(\Delta S_x)_{\ell,1} = B_1\,z_{\ell,1}\,(S_x)_1 + C_1\,(S_x)_1 = (B_1\,z_{\ell,1} + C_1)\,(S_x)_1 \tag{3-24}$$

As similarly

$$(\Delta S_y)_{2,1} = (B_1\,z_{2,1} + C_1)\,(S_y)_1 \tag{3-25}$$

$$(\Delta S_y)_{3,1} = (B_1\,z_{3,1} + C_1)\,(S_y)_1 \tag{3-26}$$

$$(\Delta S_y)_{\ell,1} = (B_1\,z_{\ell,1} + C_1)\,(S_y)_1 \tag{3-27}$$

3.3.3 Determination of Stress in the ℓth Layer Due to the Removal of the mth Layer

Changes in the average residual stress in the 2nd, 3rd, and ℓth layer due to the removal of layer No. 1 have been found in Paragraph 3.3.2. Now it is necessary to find the changes in the average residual stress in the 3rd, 4th, and ℓth layers due to the removal of layer 2, 3, and subsequent layers. This is found in a manner similar to Paragraph 3.3.2 as follows:

Let

$(\Delta S_x)_{3,2}$ = change in stress in the 3rd layer due to the removal of the 2nd layer.

$(\Delta S_x)_{4,2}$ = change in stress in the 4th layer due to the removal of the 2nd layer.

$(\Delta S_x)_{\ell,m}$ = change in stress in the ℓth layer due to the removal of the mth layer.

The values of $(\Delta S_x)_{3,2}$, $(\Delta S_x)_{4,2}$ and $(\Delta S_x)_{\ell,m}$ may be immediately written by their similarity to Equation 3-21 as follows:

$$(\Delta S_x)_{3,2} = \frac{6\ (S_x)_2\ (t_1 - t_2)\ t_1\ z_{3,2}}{t_2^{\ 3}} + \frac{(S_x)_2\ (t_1 - t_2)}{t_2} \tag{3-28}$$

Let

$$B_2 = \frac{6\ (t_1 - t_2)\ t_1}{t_2^{\ 3}} \quad \text{and} \quad C_2 = \frac{t_1 - t_2}{t_2}$$

Define

$$B_m = \frac{6\ (t_{m-1} - t_m)\ t_{m-1}}{t_m^{\ 3}} \quad \text{and} \quad C_m = \frac{t_{m-1} - t_m}{t_m}$$

Then

$$(\Delta S_x)_{3,2} = B_2\ z_{3,2}\ (S_x)_2 + C_2\ (S_x)_2 = (B_2\ z_{3,2} + C_2)\ (S_x)_2 \tag{3-29}$$

$$(\Delta S_x)_{4,2} = B_2\ z_{4,2}\ (S_x)_2 + C_2\ (S_x)_2 = (B_2\ z_{4,2} + C_2)\ (S_x)_2 \tag{3-30}$$

$$(\Delta S_x)_{\ell,m} = B_m\ z_{\ell,m}\ (S_x)_m + C_m\ (S_x)_m = (B_m\ z_{\ell,m} + C_m)\ (S_x)_m \tag{3-31}$$

Similarly

$$(\Delta S_y)_{3,2} = (B_2 z_{3,2} + C_2)(S_y)_2 \qquad (3\text{-}32)$$

$$(\Delta S_y)_{4,2} = (B_2 z_{4,2} + C_2)(S_y)_2 \qquad (3\text{-}33)$$

$$(\Delta S_x)_{\ell,m} = (B_m z_{\ell,m} + C_m)(S_y)_m \qquad (3\text{-}34)$$

3.3.4 Determination of the Total Non-Linear Stress in Any Layer

Now let the total average residual stress in the x-direction in the 1st, 2nd, 3rd, ℓth and nth layers be denoted by $(S_x)_{1_T}$, $(S_x)_{2_T}$, $(S_x)_{3_T}$, $(S_x)_{\ell_T}$ and $(S_x)_{n_T}$ then:

$$(S_x)_{1_T} = (S_x)_1 \qquad (3\text{-}35)$$

$$(S_x)_{2_T} = (S_x)_2 - (\Delta S_x)_{2,1} \qquad (3\text{-}36)$$

$$(S_x)_{3_T} = (S_x)_3 - (\Delta S_x)_{3,1} - (\Delta S_x)_{3,2} \qquad (3\text{-}37)$$

$$(S_x)_{\ell_T} = (S_x)_\ell - (\Delta S_x)_{\ell,1} - (\Delta S_x)_{\ell,2} - \dots - (\Delta S_x)_{\ell,m} - \dots - (\Delta S_x)_{\ell,m=\ell-1} \qquad (3\text{-}38)$$

$$(S_x)_{n_T} = - (\Delta S_x)_{n,1} - (\Delta S_x)_{n,2} - \dots - (\Delta S_x)_{n,m} - \dots - (\Delta S_x)_{n,m} - n-1 \qquad (3\text{-}39)$$

Similarly

$$(S_y)_{1_T} = (S_y)_1 \qquad (3\text{-}40)$$

$$(S_y)_{2_T} = (S_y)_2 - (\Delta S_y)_{2,1} \qquad (3\text{-}41)$$

$$(S_y)_{3_T} = (S_y)_3 - (\Delta S_y)_{3,1} - (\Delta S_y)_{3,2} \tag{3-42}$$

$$(S_y)_{\ell T} = (S_y)_\ell - (\Delta S_y)_{\ell,1} - (\Delta S_y)_{\ell,2} - \dots - (\Delta S_y)_{\ell,m} - \dots - (\Delta S_y)_{\ell,m=\ell-1} \tag{3-43}$$

$$(S_y)_{n_T} = -(\Delta S_y)_{n,1} - (\Delta S_y)_{n,2} - \dots - (\Delta S_y)_{n,m} - \dots - (\Delta S_y)_{n,m=n-1} \tag{3-44}$$

It is noted that $(S_x)_{n_T}$ and $(S_y)_{n_T}$ are composed completely of changes in stress due to the removal of other layers, i.e., after the n-1 layer is removed it is assumed that no stress remains in the nth layer. This is in accordance with the assumption that each layer has a uniform stress across it's section. The nth, last, layer can not have a uniform stress across its section and remain in equilibrium.

3.4 Summary

a. Cut an element of proper length and width, reference Paragraph 6, from the structure in which the non-linear residual stresses are to be determined.

b. Apply a two element strain gage to one surface, one element in the x-direction and one element in the y-direction.

c. Read $(\epsilon_x)_o$, $(\epsilon_y)_o$ and measure t_o.

d. Remove 1st layer from the opposite surface and read $(\epsilon_x)_1$, $(\epsilon_y)_1$ and measure t_1. The thickness of the layers removed is arbitrary but it will determine the accuracy of the final stress distribution. It is suggested that the thickness of a removed layer be equal to or less then 1/12 of the section height for good results. The smaller the thickness of the removed layer the more accurate the results will be.

e. Determine $(\Delta\epsilon_x)_1$ and $(\Delta\epsilon_y)_1$ from Equations 3-3 and 3-4 with ℓ=1.

f. Determine $(S_x)_1$ and $(S_y)_1$ by Equations 3-18 and 3-19 with ℓ= 1. This is the average residual stress which existed in 1st layer before removal of the first layer, reference Equations 3-35 and 3-40.

g. Remove 2nd layer and read $(\epsilon_x)_2$, $(\epsilon_y)_2$ and measure t_2.

h. Determine $(\Delta\epsilon_x)_2$ and $(\Delta\epsilon_y)_2$ from Equations 3-3 and 3-4 with $\ell = 2$.

i. Determine $(S_x)_2$ and $(S_y)_2$ by Equations 3-18 and 3-19 with $\ell = 2$.

j. Determine $(\Delta S_x)_{2,1}$ and $(\Delta S_y)_{2,1}$ by Equations 3-31 and 3-34 with $\ell = 2$ and $m = 1$.

k. Determine $(S_x)_{2_T}$ and $(S_y)_{2_T}$ by Equations 3-38 and 3-43 with $\ell = 2$ and $m = 1$. This is the average residual stress which existed in 2nd layer before removal of the first and second layer.

l. Remove 3rd layer and read $(\epsilon_x)_3$, $(\epsilon_y)_3$ and measure t_3.

m. Determine $(\Delta\epsilon_x)_3$ and $(\Delta\epsilon_y)_3$ by Equations 3-3 and 3-4 with $\ell = 3$.

n. Determine $(S_x)_3$ and $(S_y)_3$ by Equations 3-18 and 3-19 with $\ell = 3$.

o. Determine $(\Delta S_x)_{3,1}$ and $(\Delta S_y)_{3,1}$ by Equations 3-31 and 3-34 with $\ell = 3$ and $m = 1$.

p. Determine $(\Delta S_x)_{3,2}$ and $(\Delta S_y)_{3,2}$ by Equations 3-31 and 3-34 with $\ell = 3$ and $m = 2$.

q. Determine $(S_x)_{3_T}$ and $(S_y)_{3_T}$ by Equations 3-38 and 3-43 with $\ell = 3$ and m has progressive values beginning at 1 and going to $\ell - 1$ which in this case is 2.

r. Remove additional layers of material from the specimen until only one layer is left. After the removal of each layer read $(\epsilon_x)_\ell$, $(\epsilon_y)_\ell$ and measure t_ℓ.

s. Determine successive values of $(\Delta\epsilon_x)_\ell$ and $(\Delta\epsilon_y)_\ell$ by Equations 3-3 and 3-4.

t. Determine successive values of $(S_x)_\ell$ and $(S_y)_\ell$ by Equations 3-18 and 3-19.

u. Determine successive values of $(\Delta S_x)_{\ell,1}$, $(\Delta S_x)_{\ell,2}$, $(\Delta S_x)_{\ell,3}$, ... $(\Delta S_x)_{\ell,m = \ell-1}$, after the removal of each layer by Equation 3-31. Follow similar procedure for changes in stress in y-direction by using Equation 3-34.

v. Determine successive values of $(S_x)_{\ell\, T}$ and $(S_y)_{\ell\, T}$ by Equations 3-38 and 3-43.

w. Determine the value of $(S_x)_{n_T}$ and $(S_y)_{n_T}$, the residual stress in the last layer which existed before the removal of any layers, by Equations 3-39 and 3-44.

3.4.1 Laboratory and Computing Work Involved

The steps (a) through (w) outlined in Paragraph 3.4 are a summary of the steps required to determine the non-linear stress distribution in a structural element. However, the only laboratory work involved is in obtaining the element from the structure, applying the two strain gages, taking the strain gage readings before and after the removal of each layer and measuring the thickness of the specimen before and after the removal of each layer. This data plus E_x, E_y, μ_{xy}, μ_{yx} is recorded on the load sheet shown in Figure 5-1 . From the load sheet IBM cards are punched. These cards are placed behind the program desk and fed into an IBM 7094 computer. The 7094 prints a magnetic tape which is later transferred to a Stromberg Carlson 4020 unit to obtain vellum or microfilm type graphical output. Either may be used to produce data as shown in Figure 5-2 or 5-3. Printed tabular data may also be obtained as shown in Table 5-2 or 5-4.

4. TOTAL RESIDUAL STRESS DISTRIBUTION DETERMINED BY REMOVAL OF ELEMENT FROM PART AND REMOVAL OF LAYERS FROM ELEMENT

The linear residual stress distribution caused by removal of the element from the part has been determined in Paragraph 2, resulting in Equations 2-7 and 2-8.

The non-linear residual stress distribution caused by removal of layers from the element has been determined in Paragraph 3, resulting in Equations 3-35 through 3-44. The final step is to determine the total residual stress in any layer, thus

$$(S_x)_{\ell_{TOTAL}} = (S_x)_{\ell_{M+A}} + (S_x)_{\ell_T} \tag{4-1}$$

$$(S_y)_{\ell_{TOTAL}} = (S_y)_{\ell_{M+A}} + (S_y)_{\ell_T} \tag{4-2}$$

Since the last layer, designated as layer n, is treated as a special case in Paragraph 3 the equations for the last layer become:

$$(S_x)_{n_{TOTAL}} = (S_x)_{n_{M+A}} + (S_x)_{n_T} \tag{4-3}$$

$$(S_y)_{n_{TOTAL}} = (S_y)_{n_{M+A}} + (S_y)_{n_T} \tag{4-4}$$

5. FORTRAN IV PROGRAM DESCRIPTION

A Fortran IV program has been written which solves the equations shown in Paragraphs 2, 3 and 4. A list of the program cards in Fortran IV language is given in Reference 2. The list includes all card punch instructions for the computing machine including output format and graphical display statements.

A specially designed load sheet is used in connection with this program. A sample is shown in Figure 5-1.

5.1 Tabular Output

If no linear strains were entered in the load sheet, the tabular output will be as shown for sample problem 1 in Table 5-2. If linear strains were entered in the load sheet the tabular output will be as shown for sample problem 2 in Table 5-4.

5.1.1 Equilibrium Check

An equilibrium check of the layer removal process is given in the tabular output, reference Tables 5-2 and 5-4. The sum of the forces in all of the layers in both the X and Y directions, i.e., SFX and SFY, should be equal to zero if all strain gage and thickness readings taken during the layer removal process are exact. Similarly the sum of the moments about the neutral axis, i.e., SMX and SMY, caused by all the layers should be equal to zero. Due to slight errors in measurement SFX, SFY, SMX and SMY will seldom be precisely zero. Ordinarily these values will be within 5 percent of the maximum positive or negative forces and moments which are listed in the output directly under the values of SFX, SFY, SMX and SMY. If the euqilibrium check indicates more than 10% error it may be desirable to repeat the experiment.

5.2 Graphical Display

Graphical display of the residual stress distribution by the SC-4020 machine is included in the program. It may be called for or not as indicated by note 4 on the load sheet, reference Figure 5-1. Examples of the graphical display output are shown for sample problem 1 and 2 in Figures 5-2 and 5-3.

Form 60-1641 (6-65)

SAMPLE LOAD SHEET

PAGE ______ OF ______

PROGRAM NO G296: TOTAL RESIDUAL STRESS DISTRIBUTION BY STRAIN RELIEF CAUSED BY REMOVAL OF ELEMENT FROM PART AND REMOVAL OF LAYERS FROM ELEMENT

TITLE CARD

NAME	DATE	CENTER SUBTITLE

DATA CARDS

μ_{yx}	μ_{xy}	E_x	E_y	NT	IP	CENTER UNCUT SURFACE NAME	I	NO

STRAIN DUE TO RELEASE OF MOMENT AND AXIAL LOAD

$(\epsilon_x)_{OSM}$	$(\epsilon_x)_{ISM}$	$(\epsilon_y)_{OSM}$	$(\epsilon_y)_{ISM}$	K

A TITLE CARD AND ALL THE DATA CARDS SHOWN ARE REQUIRED FOR EVERY CASE, IN THE ORDER LISTED.
THE NUMBER OF SUBSCRIPT ℓ CARDS REQUIRED IS ONE PLUS THE NO OF LAYERS REMOVED. 100 MAXIMUM

LAYER REMOVAL DATA

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$

LAYER REMOVAL DATA – CONTINUED

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$

NOTES
1. NT TOTAL NO. OF LAYERS, I.E., LAYERS REMOVED PLUS LAST LAYER.
2. I 1 IF ANOTHER CASE FOLLOWS. ENTER 0 OR LEAVE BLANK FOR THE LAST CASE.
3. NO CASE NUMBER, AN INTEGER
4. IP * FOR CURVE PLOT. 0 IF NO PLOT DESIRED
   * 1 35 MM, 2 HARD COPY, 3 BOTH WITH OFFLINE LIST
   4 35 MM, 5 HARD COPY, 6 BOTH NO OFFLINE LIST
5. A DECIMAL POINT MUST BE INCLUDED IN ALL VALUES EXCEPT NT, IP, I, K AND NO
6. IF STRAIN DUE TO THE RELEASE OF MOMENT AND AXIAL LOAD WAS NOT FOUND, USE A BLANK CARD.
7. K 0 FOR TOTAL PLOT ONLY, 1 FOR BOTH TOTAL AND LAYER REMOVAL PLOTS.

FIGURE 5-1

The program includes the following automatic operations which are displayed on hard copy (vellum) and/or 35 MM microfilm:

a. A separate vertical scale in psi is selected for the X and Y-stress to fit the maximum value of residual stress, computed for the particular problem, into the available vertical space. Horizontal grid lines are drawn and labeled in psi.

b. A horizontal scale is selected based on the original thickness of the element and the available horizontal space. Vertical grid lines are drawn and labeled in inches.

c. A scaled drawing showing the element and each layer removed is placed horizontally under the grid pattern formed by (a) and (b). The scale is the same as horizontal grid scale. The element layers are numbered and the uncut surface is labeled for orientation purposes. A dimension is given for the original thickness of the element.

d. Points are plotted showing the computed X and Y-stress at horizontal stations corresponding to the mid-point of each layer removed.

e. Straight lines are drawn to connect the plotted points.

f. The page is titled with "Residual Stress Calculated by Removal of Layers from Element" if the values of $(\epsilon_x)_{OSM}$, $(\epsilon_x)_{ISM}$, $(\epsilon_y)_{OSM}$ and $(\epsilon_y)_{ISM}$ were zero. If any of these values were other than zero then the title would also include "Plus Residual Stress due to Removal of Element from Part."

g. The following information will also appear on the page if it was entered on the load sheet. Name of individual who prepared it, the date, the subtitle, the case number, μ_{xy}, μ_{yx}, E_x and E_y.

h. Paper cut lines are shown at the four corners of the page with a note to make the long dimension 11 inches. The short dimension will then be 8-1/2 inches.

5.3 Sample Problems

Two sample problems are included. Each problem consists of a load sheet, tabular output and graphical display. Reference Tables 5-1 and 5-2 and Figure 5-2 for Problem 1 and Tables 5-3 and 5-4 and Figure 5-3 for Problem 2. The data for the sample problems is that obtained from Specimen 2 mentioned in Paragraph 7. The same layer removal data is used for both problems. In Problem 1 no strains were listed for $(\epsilon_x)_{OSM}$, $(\epsilon_x)_{ISM}$, $(\epsilon_y)_{OSM}$ and $(\epsilon_y)_{ISM}$ since there was no release of moment and axial load involved. However, when the ends of the bar were cut off to obtain the 7-1/2 inch specimen described in Paragraph 7.1, very slight strains were noted for these values. These values are not truly due to the release of moment and axial load but they were listed as such for Problem 2 in order to show the different type of output obtained when linear strains due to the release of moment and axial load are included. In each problem a 35 millimeter film, which could be enlarged to show the same data as in Figures 5-2 and 5-3, was obtained as part of the output. The microfilm is not shown in this report.

5.4 User Instructions

The following information is intended to be of help to those interested in determining residual stresses by the method outlined in this report.

5.4.1 Specimen

The method is applicable only if the specimen (or element) which is to be cut from the structure is of constant or nearly constant thickness and any layer of material parallel to the top and bottom surface is the same material as every other layer. The specimen may be round, square or rectangular in planform, and can be slightly curved, as from a shell or cylinder, but the minimum dimension should be equal to or greater than twice the thickness plus the gage length of the strain gage.

5.4.2 Strain Gages

A strain gage is applied to one surface in the X-direction and another gage is applied in the Y-direction. Corresponding gages are applied to the opposite surface. Suitable dummy gages, applied to a piece of unstressed

TABLE 5-1

LOAD SHEET FOR SAMPLE PROBLEM 1

Form 53-1642 (6-65)

PAGE 1 OF 2

PROGRAM NO. C296. TOTAL RESIDUAL STRESS DISTRIBUTION BY STRAIN RELIEF CAUSED BY REMOVAL OF ELEMENT FROM PART AND REMOVAL OF LAYERS FROM ELEMENT

TITLE CARD

NAME	DATE	CENTER SUBTITLE
G.A. GURTMAN	JULY 8 1965	SPECIMEN NO. 2

DATA CARDS

μ_{yx}	μ_{xy}	E_x	E_y	NT	IP	CENTER UNIQUE SURFACE NAME	NO
0.310	0.310	10500000.0	10500000.0	12	3	TOP SIDE	

STRAIN DUE TO RELEASE OF MOMENT AND AXIAL LOAD

$(\epsilon_x)_{OSM}$	$(\epsilon_x)_{ISM}$	$(\epsilon_y)_{OSM}$	$(\epsilon_y)_{ISM}$	K

A TITLE CARD AND ALL THE DATA CARDS SHOWN ARE REQUIRED FOR EVERY CASE, IN THE ORDER LISTED. THE NUMBER OF SUBSCRIPT ℓ CARDS REQUIRED IS ONE PLUS THE NO. OF LAYERS REMOVED. 100 MAXIMUM

LAYER REMOVAL DATA

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$
3.053	0.000042	0.000025
2.756	-0.000201	0.000126
2.519	-0.000359	0.000160
2.266	-0.000377	0.000132
2.002	0.000048	0.000007
1.765	0.000595	-0.000157
1.513	0.001105	-0.000294
1.268	0.001373	-0.000427
1.013	0.001565	-0.000454
0.761	0.001520	-0.000479
0.510	0.001483	-0.000469
0.259	0.001421	-0.000515

LAYER REMOVAL DATA CONTINUED

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$

NOTES

1. NT TOTAL NO. OF LAYERS, I.E., LAYERS REMOVED PLUS LAST LAYER.
2. [illegible] 1 IF ANOTHER CASE FOLLOWS, ENTER 0 OR LEAVE BLANK FOR THE LAST CASE.
3. NO CASE NUMBER, AN INTEGER.
4. IP [illegible] FOR CURVE PLOT, [illegible] NO PLOT DESIRED.
   1 35 MM, 2 HARD COPY, 3 BOTH, WITH OFFLINE LIST
   4 35 MM, 5 HARD COPY, 6 BOTH, NO OFFLINE LIST
5. A DECIMAL POINT MUST BE INCLUDED IN ALL VALUES EXCEPT NT, IP, I, K AND NO.
6. IF STRAIN DUE TO THE RELEASE OF MOMENT AND AXIAL LOAD WAS NOT FOUND, USE A BLANK CARD.
7. K 0 FOR TOTAL PLOT ONLY, 1 FOR BOTH TOTAL AND LAYER REMOVAL PLOTS.

TABLE 5-2
TABULAR OUTPUT FOR SAMPLE PROBLEM I

PREPARED BY G.A. GURTMAN

DATE JULY 8 1965

RESIDUAL STRESS DETERMINATION BY STRAIN GAGES AND MATERIAL REMOVAL

SPECIMEN NO.2

MU-YX	MU-XY	E-X	E-Y	NO. LAYERS	CASE NO.
0.3100	0.3100	10500000.	10500000.	12	1

LAYER	THICKNESS OF SPECIMEN	X STRAIN READING	Y STRAIN READING	TOTAL RESIDUAL STRESS-X	TOTAL RESIDUAL STRESS-Y
	3.0530	0.0000420	0.0000250		
1	2.7560	-0.0002010	0.0001260	9821.72	-1191.00
2	2.5190	-0.0003590	0.0001600	3703.90	1328.60
3	2.2660	-0.0003770	0.0001320	-5425.86	1637.28
4	2.0020	0.0000480	0.0000070	-20176.11	-795.14
5	1.7650	0.0005950	-0.0001570	-15129.36	-521.34
6	1.5130	0.0011050	-0.0002940	-1270.73	-738.07
7	1.2680	0.0013730	-0.0004270	12374.31	1440.97
8	1.0130	0.0015650	-0.0004540	13336.60	-1206.66
9	0.7610	0.0015200	-0.0004790	13265.90	567.21
10	0.5100	0.0014830	-0.0004690	4011.99	-443.71
11	0.2590	0.0014210	-0.0005150	-3960.66	78.95
12				-11498.80	182.87

EQUILIBRIUM CHECK

SFX = 91.99	SFY = -1.46	SMX = -338.60	SMY = 20.38
+SFX = 14577.43	+SFY = 1290.80	+SMX = 10162.27	+SMY = 926.56
-SFX =-14485.44	-SFY = -1292.27	-SMX =-10500.88	-SMY = -906.17

GRAPHICAL DISPLAY FOR SAMPLE PROBLEM 1

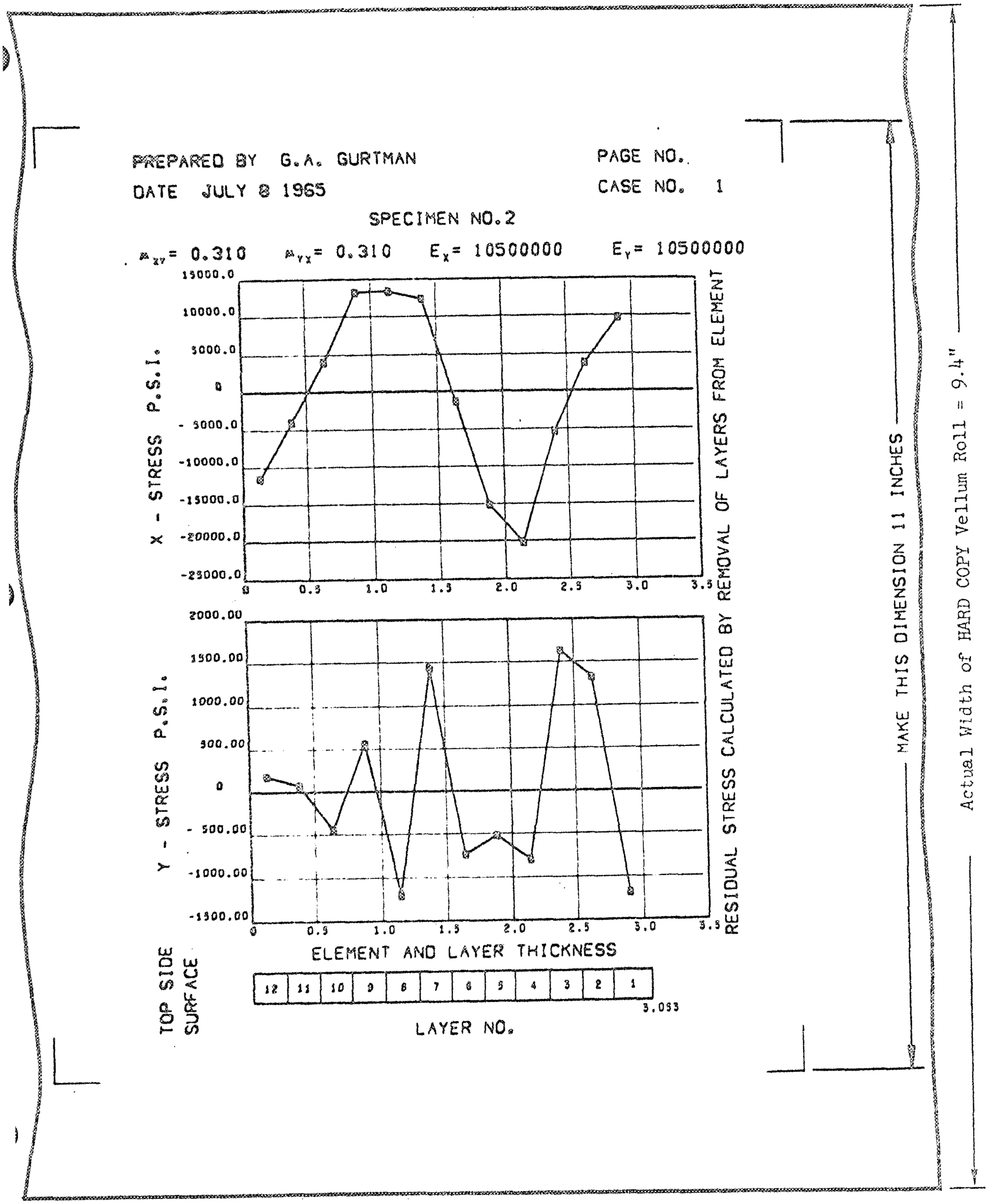


FIGURE 5-2

TABLE 5-3
LOAD SHEET FOR SAMPLE PROBLEM 2

Form 60-1641 (6-65)

PAGE 2 OF 2

PROGRAM NO G296: TOTAL RESIDUAL STRESS DISTRIBUTION BY STRAIN RELIEF CAUSED BY REMOVAL OF ELEMENT FROM PART AND REMOVAL OF LAYERS FROM ELEMENT

TITLE CARD

NAME	DATE	CENTER SUBTITLE
G. A. GURTMAN	JULY 8 1965	SPECIMEN NO. 2

DATA CARDS

μ_{yx}	μ_{xy}	E_x	E_y	NT	IP	CENTER UNCUT SURFACE NAME	I	NO
0.310	0.310	10500000.0	10500000.0	12	3	TOP SIDE		2

STRAIN DUE TO RELEASE OF MOMENT AND AXIAL LOAD

$(\epsilon_x)_{OSM}$	$(\epsilon_x)_{ISM}$	$(\epsilon_y)_{OSM}$	$(\epsilon_y)_{ISM}$	K
0.000042	0.000047	0.000025	0.000052	1

A TITLE CARD AND ALL THE DATA CARDS SHOWN ARE REQUIRED FOR EVERY CASE, IN THE ORDER LISTED

THE NUMBER OF SUBSCRIPT ℓ CARDS REQUIRED IS ONE PLUS THE NO OF LAYERS REMOVED. 100 MAXIMUM

LAYER REMOVAL DATA

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$
3.053	0.000042	0.000025
2.756	-0.000201	0.000126
2.519	-0.000359	0.000160
2.266	-0.000377	0.000132
2.002	0.000048	0.000007
1.765	0.000595	-0.000157
1.513	0.001105	-0.000294
1.268	0.001373	-0.000427
1.013	0.001565	-0.000454
0.761	0.001520	-0.000479
0.510	0.001483	-0.000469
0.259	0.001421	-0.000515

LAYER REMOVAL DATA - CONTINUED

t_ℓ	$(\epsilon_x)_\ell$	$(\epsilon_y)_\ell$

NOTES

1. NT TOTAL NO. OF LAYERS I.E., LAYERS REMOVED PLUS LAST LAYER.
2. I 1 IF ANOTHER CASE FOLLOWS, ENTER 0 OR LEAVE BLANK FOR THE LAST CASE.
3. NO CASE NUMBER AN INTEGER.
4. IP 1 FOR CURVE PLOT, 0 IF NO PLOT DESIRED.
   - 1 35 MM, 2 HARD COPY, 3 BOTH WITH OFFLINE LIST
   - 4 35 MM, 5 HARD COPY, 6 BOTH - NO OFFLINE LIST
5. A DECIMAL POINT MUST BE INCLUDED IN ALL VALUES EXCEPT NT, IP, I, K AND NO.
6. IF STRAIN DUE TO THE RELEASE OF MOMENT AND AXIAL LOAD WAS NOT FOUND, USE A BLANK CARD.
7. K 0 FOR TOTAL PLOT ONLY, 1 FOR BOTH TOTAL AND LAYER REMOVAL PLOTS.

TABLE 5-4
TABULAR OUTPUT FOR SAMPLE PROBLEM 2

PREPARED BY G.A. GERTMAN

DATE JULY 8 1965

TOTAL RESIDUAL STRESS BY STRAIN RELIEF CAUSED BY REMOVAL OF ELEMENT FROM PART AND REMOVAL OF LAYERS FROM ELEMENT

SPECIMEN NO.2

MU-YX	MU-XY	E-X	E-Y	NO. LAYERS	CASE NO.
0.3100	0.3100	10500000.	10500000.	12	2

LINEAR RESIDUAL STRAINS

TOP SIDE SURFACE X - DIRECTION	TOP SIDE SURFACE Y - DIRECTION	OTHER SURFACE X - DIRECTION	OTHER SURFACE Y - DIRECTION
0.0000420	0.0000250	0.0000470	0.0000520

LAYER	THICKNESS OF SPECIMEN	STRAIN READING FROM LAYER REMOVAL X	STRAIN READING FROM LAYER REMOVAL Y	RESIDUAL STRESS FROM LAYER REMOVAL X	RESIDUAL STRESS FROM LAYER REMOVAL Y	LINEAR RESIDUAL STRESS X	LINEAR RESIDUAL STRESS Y	TOTAL RESIDUAL STRESS X	TOTAL RESIDUAL STRESS Y
	3.0530	0.0000420	0.0000250						
1	2.7560	-0.0002010	0.0001260	9821.72	-1191.00	725.57	757.17	10547.39	-433.34
2	2.5190	-0.0003590	0.0001600	3703.90	1328.60	712.09	728.16	4415.99	2056.77
3	2.2860	-0.0003770	0.0001320	-5425.80	1637.28	699.62	701.55	-4726.23	2338.83
4	2.0020	0.0000480	0.0000070	-20176.11	-795.14	686.47	673.47	-19489.54	-121.68
5	1.7650	0.0005950	-0.0001570	-15129.36	-521.34	673.73	646.26	-14455.63	124.91
6	1.5130	0.0011050	-0.0002940	-1270.73	-738.07	661.29	619.70	-609.44	-118.37
7	1.2680	0.0013730	-0.0004270	12374.31	1440.97	648.65	592.70	13022.96	2033.68
8	1.0130	0.0015650	-0.0004540	13336.60	-1205.66	635.93	565.54	13972.53	-641.12
9	0.7610	0.0015200	-0.0004790	13265.90	567.21	623.04	538.01	13888.93	1105.21
10	0.5100	0.0014830	-0.0004690	4011.99	-443.71	610.24	510.65	4622.23	66.98
11	0.2590	0.0014210	-0.0005150	-3960.66	78.95	597.47	483.42	-3363.18	562.39
12				-11498.80	182.87	584.50	455.72	-10914.30	638.59

EQUILIBRIUM CHECK OF LAYER REMOVAL PROCESS

SFX = 91.99	SFY = -1.46	SMX = -338.60	SMY = 20.38
+SFX = 14577.43	+SFY = 1290.80	+SMX = 10162.27	+SMY = 926.56
-SFX = -14485.44	-SFY = -1292.27	-SMX = -10500.88	-SMY = -906.17

GRAPHICAL DISPLAY FOR SAMPLE PROBLEM 2

PREPARED BY G.A. GURTMAN PAGE NO.

DATE JULY 8 1965 CASE NO. 2

SPECIMEN NO.2

μ_{XY} = 0.310 μ_{YX} = 0.310 E_X = 10500000 E_Y = 10500000

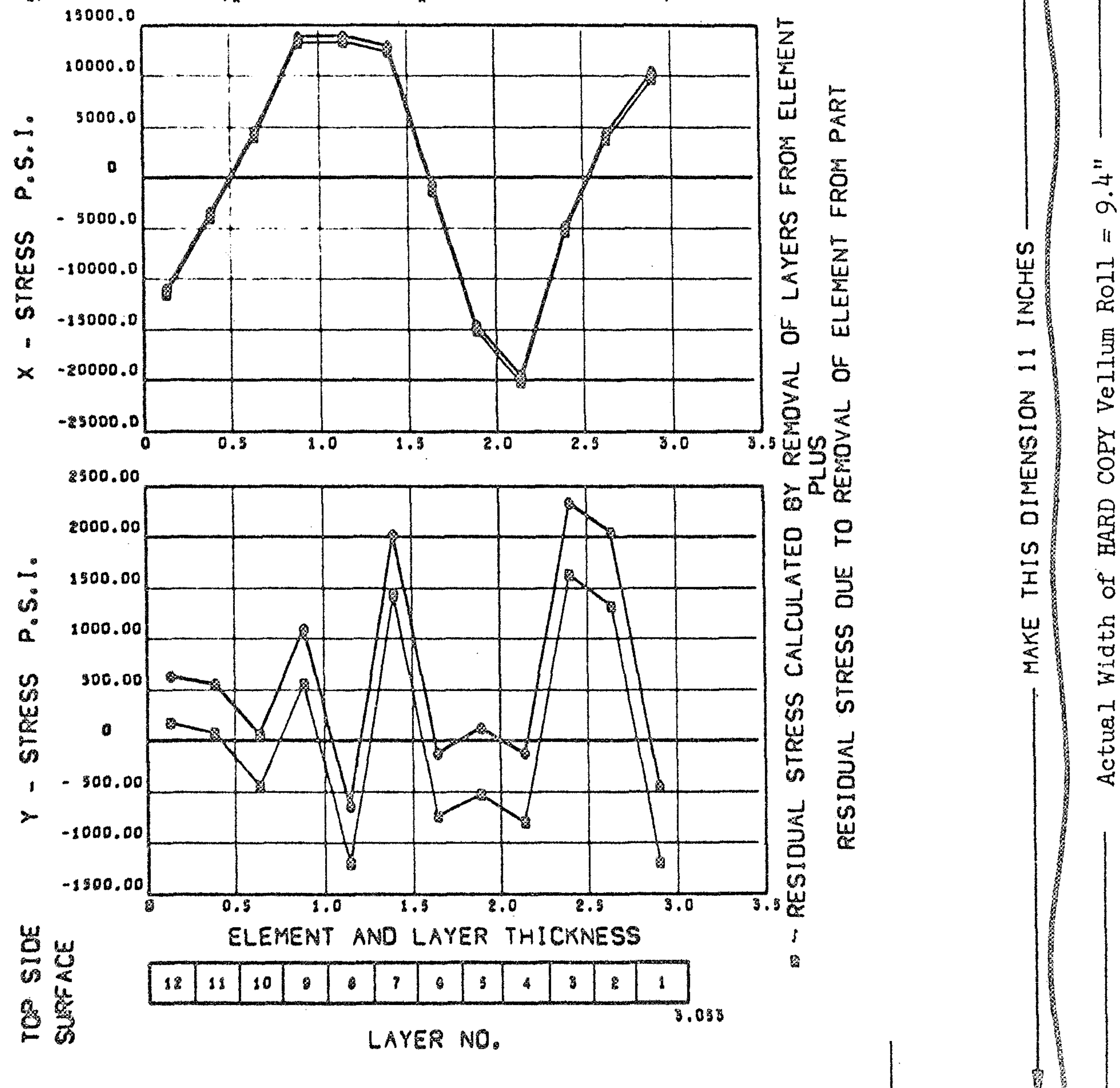


FIGURE 5-3

structure are used for temperature compensation. The gages are read before and after cutting the element from the structure. From these readings the released strains $(\epsilon_x)_{OSM}$, $(\epsilon_x)_{ISM}$, $(\epsilon_y)_{OSM}$ and $(\epsilon_y)_{ISM}$ are determined by Equations 2-1 through 2-4 and entered into the load sheet.

The strain gages are then removed from one surface of the specimen and layers of material are removed from this side. Thickness of the specimen and the strain gage readings are taken before and after the removal of each layer. These data are entered into the load sheet.

5.4.2.1 Strain Gage Length

It is mentioned in Paragraph 6 that the non-linear residual stress distribution varies in a specimen from zero at the cut edges to the true non-linear residual stress value at a distance equal to the thickness from the edge. It has been proven by the tests mentioned in Paragraph 7 that the method used in this report gives the correct value of the residual stress distribution as long as no part of the strain gage element is closer to the edge of the specimen than the thickness of the specimen. It therefore follows that whether or not the true non-linear residual stress varies along the length of the structure, or the specimen cut from it. the procedure in this report will give valid results of the residual stress with the following limitation. If the stress distribution varies over the gage length of the strain gage then the procedure will only give the average residual stress over the gage length.

In view of the foregoing discussion the selection of the most appropriate strain gage length will depend on the following considerations:

a. The effect of the strain gage on the specimen size, i.e., specimen size $\geq 2\ t_o$ plus gage length of strain gage.

b. The amount of change in residual stress distribution expected along the length of the structure, i.e., the shorter the gage length the less averaging will take place.

c. The degree of skill required to install the gage, i.e., small gages are more difficult to install than large gages.

5.4.3 Modulus of Elasticity and Poisson's Ratio

Enter E_x, E_y, μ_{xy} and μ_{yx} in the load sheet. If the material is isotropic then $E_x = E_y$ and $\mu_{xy} = \mu_{yx}$. If the material is orthotropic there may be cases where three of these values are known and the other is not. In this case the following relationship may be used to calculate the unknown value.

$$\frac{\mu_{xy}}{\mu_{yx}} = \frac{E_y}{E_x} \qquad (5\text{-}1)$$

Reference 1, Page 2.

5.4.4 Load Sheet

A sample load sheet is shown in Figure 5-1. Complete instructions for entering the data and for obtaining the desired output is contained on the sheet. Copies of the load sheet may be obtained from either of the authors.

5.4.5 To Obtain Desired Type of Output

If it is desired to have the output consist of only tabulated values of residual stress then, as indicated in Note 4 of the load sheet, enter 1P = 0 in column 58 of the first data card.

The tabular format will be as shown in Table 5-2 if there are no strains due to the release of moment and axial load, i.e., a blank card is used for the second data card as indicated in Note 6 of the data sheet. If, however, values of strain are entered in the second data card then the tabular format will be as shown in Table 5-4.

If a curve plot is desired then enter for IP, in column 58 of the first data card, the value shown in Note 4 on the load sheet. This will give the plot in the desired form. In Note 4, 35 MM means 35 millimeter microfilm. HARD COPY means a roll of vellum 9.4 inches wide with the desired curve plots on it. Each curve is 5.8 in. by 7.5 in. Tabular output (offline list) in the format shown in Tables 5-2 and 5-4 may be called for or not as indicated in Note 4 of the load sheet.

When a curve plot is desired it is possible to get only a total plot, by entering K = 0 in column 53 of the second data card, or it is possible to get both the total plot plus the layer removal data plot shown separately on the same sheet, by entering K = 1 in column 53 of the second data card. This is indicated in Note 7 on the load sheet.

5.4.6 To Obtain Residual Stress Near Surface Only

Situations may arise when it is desired to obtain the residual stress near the surface only. This could arise in a study of different types of shot peening where the residual stress distribution due to the shot peening is high close to the surface and very low in the rest of the section. In this situation it would be possible to follow the same procedure described in this report but to take off only a few very thin layers near the surface which has been shot peened. The stresses obtained will be correct on the thin layers removed. The stress shown for the remainder of the section will be a constant stress of a low magnitude. This constant low stress would not be absolutely accurate but this would be of no consequence since it would be the stresses near the surface that are of interest in the study. This is mentioned merely to point out that it is not necessary to remove layers through the whole section if only the stresses in a part of it are of interest.

6. EXPERIMENTAL DETERMINATION OF MINIMUM ELEMENT SIZE

6.1 Photostress Method

A number of tests were performed to determine the minimum length to thickness ratio of an element required to leave the residual stress distribution in that element essentially undisturbed. Two seperate methods of approach were used. Firstly, a 2014-T6 aluminum bar 1 x 1/2 x 27 in. was stretched in a tensile testing machine to a strain of approximately 0.008 in/in. This effectively relieves the bar of any initial residual stresses without causing any noticeable changes in the bar's dimensions. The bar was then placed in a jig similar to the one depicted in Figure 6-1, and subjected to a bending moment of 8,120 in-lbs. This moment was sufficient to cause considerable plastic deformation of the specimen. Following this, 0.119 in. thick photostress material (Budd Co. Type S) was bonded to the central 12 inches of the bar, and a grid was scribed on the photostress. A 10 inch element was cut from this coated area, and shear strains were recorded at points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 as shown in Figure 6-2. Slices were then progressively removed from each end of the specimen, and shear strain readings taken after each cut, with a Budd Co. SF/Z-U small field reflection polarsicope. A fringe pattern photograph of the specimen after a number of such slices were removed is shown in Figure 6-3.

The resulting data is shown plotted as a function of the specimens length to thickness ratio in Figures 6-4 through 6-8. The apparently unusual behavior of the shear strains at points 1, 2, 10, and 11 is due to the appearance of "time-edge" effect in the photostress material. This phenomena, caused by the absorbtion of water vapor in the photostress plastic, gives rise to compressive fringes. When these fringes are superimposed on those caused by strains transferred from the aluminum bar, large errors in strain measurement ensue. This "time-edge" fringe is clearly visible in Figure 6-3, as a light band on the top and bottom surfaces of the central section of the bar. A definite trend is noted from the data gathered at points 3, 4, 5, 6, 7, 8 and 9, however. These data indicate that so long as the specimen's length is at least twice its thickness, the strain distribution at the center of the specimen remains undisturbed by cuts at either end.

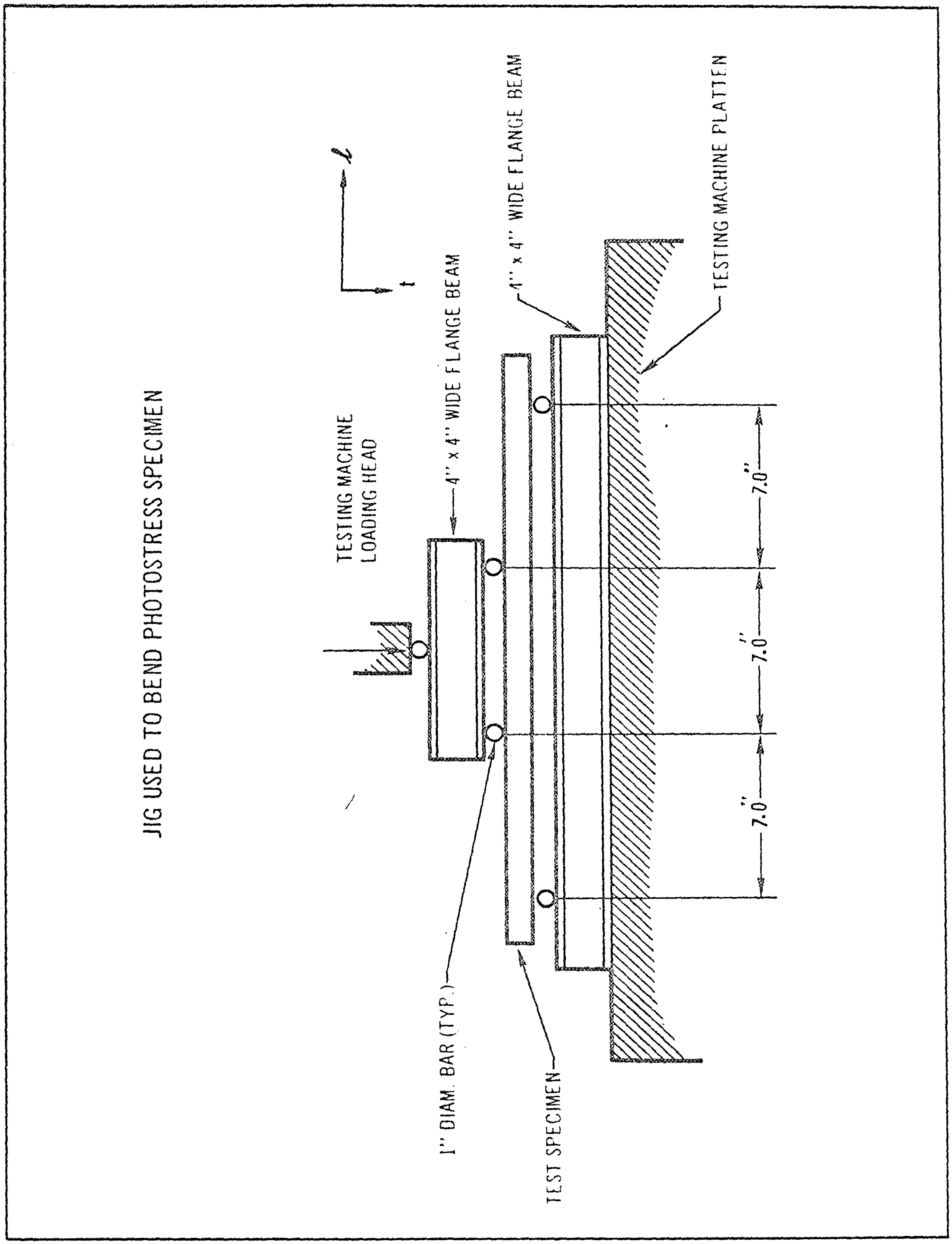

JIG USED TO BEND PHOTOSTRESS SPECIMEN
TESTING MACHINE LOADING HEAD
4'' x 4'' WIDE FLANGE BEAM
4'' x 4'' WIDE FLANGE BEAM
TESTING MACHINE PLATTEN
1'' DIAM. BAR (TYP.)
TEST SPECIMEN
7.0''
7.0''
7.0''
l
t

FIGURE 6-1

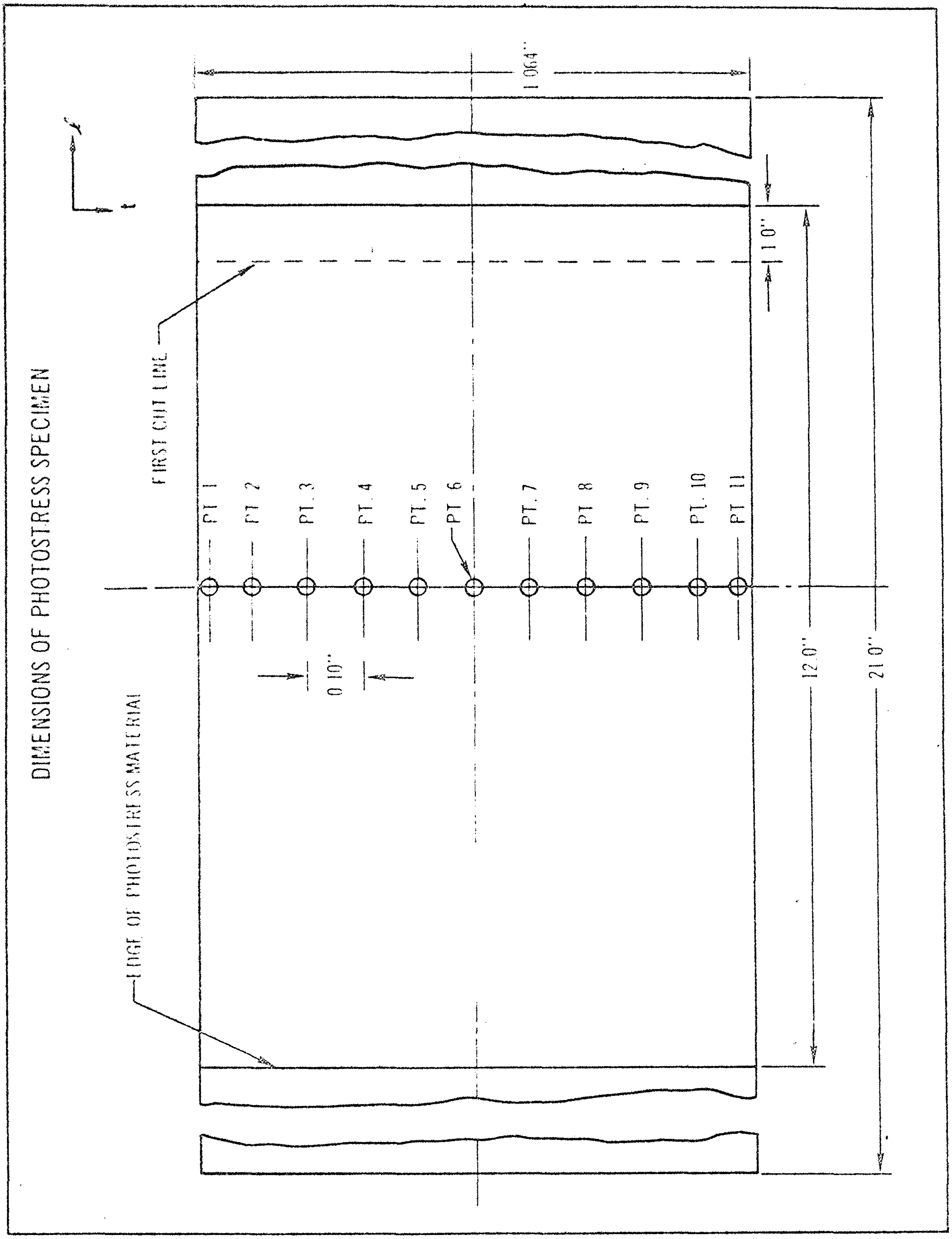
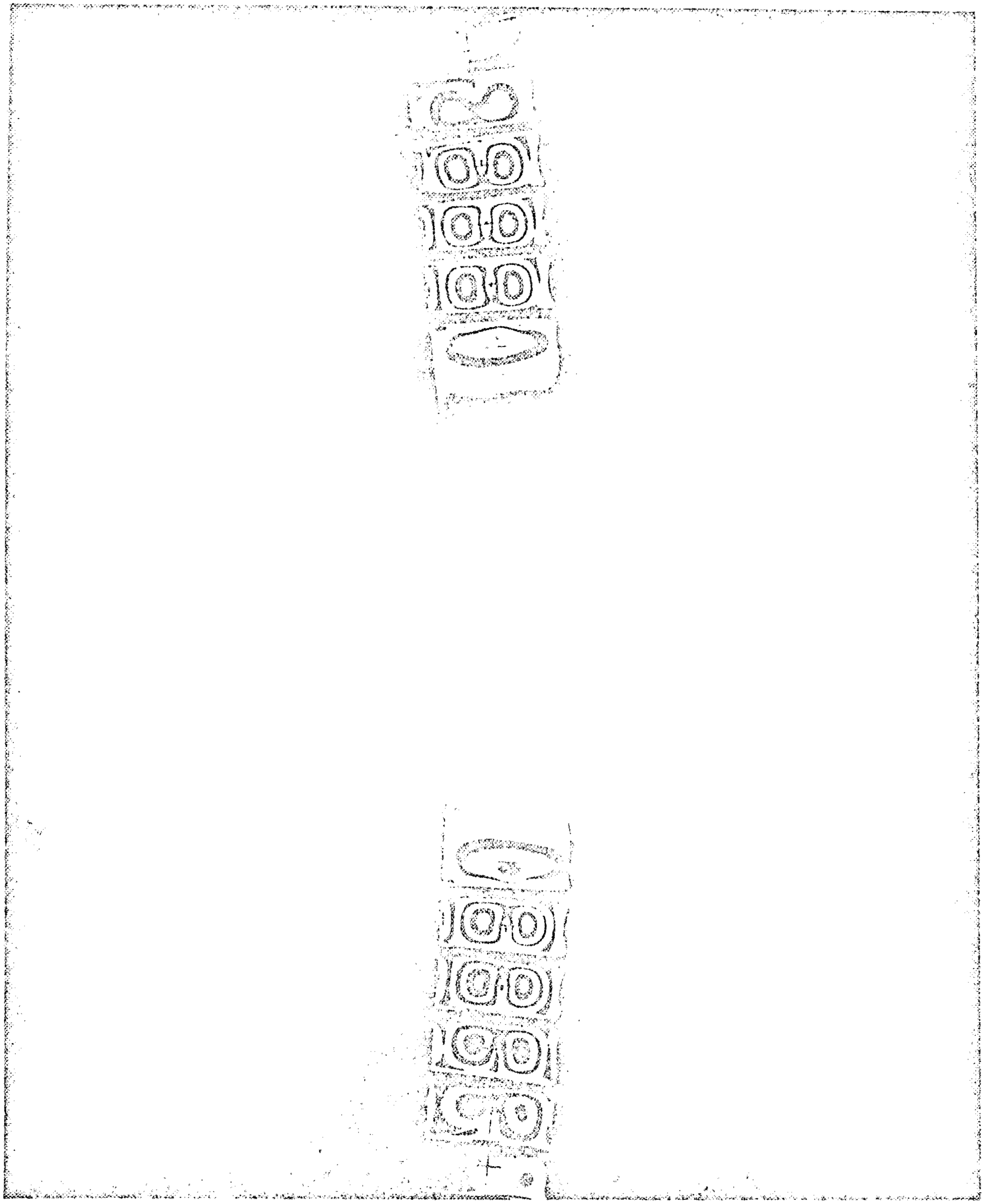

DIMENSIONS OF PHOTOSTRESS SPECIMEN
FIRST CUT LINE
EDGE OF PHOTOSTRESS MATERIAL
PT. 1
PT. 2
PT. 3
PT. 4
PT. 5
PT. 6
PT. 7
PT. 8
PT. 9
PT. 10
PT. 11
0.10''
1.064''
1.0''
12.0''
21.0''

FIGURE 6-2



FRINGE PATTERN DURING SLICE REMOVAL PROCEDURE

FIGURE 6-3

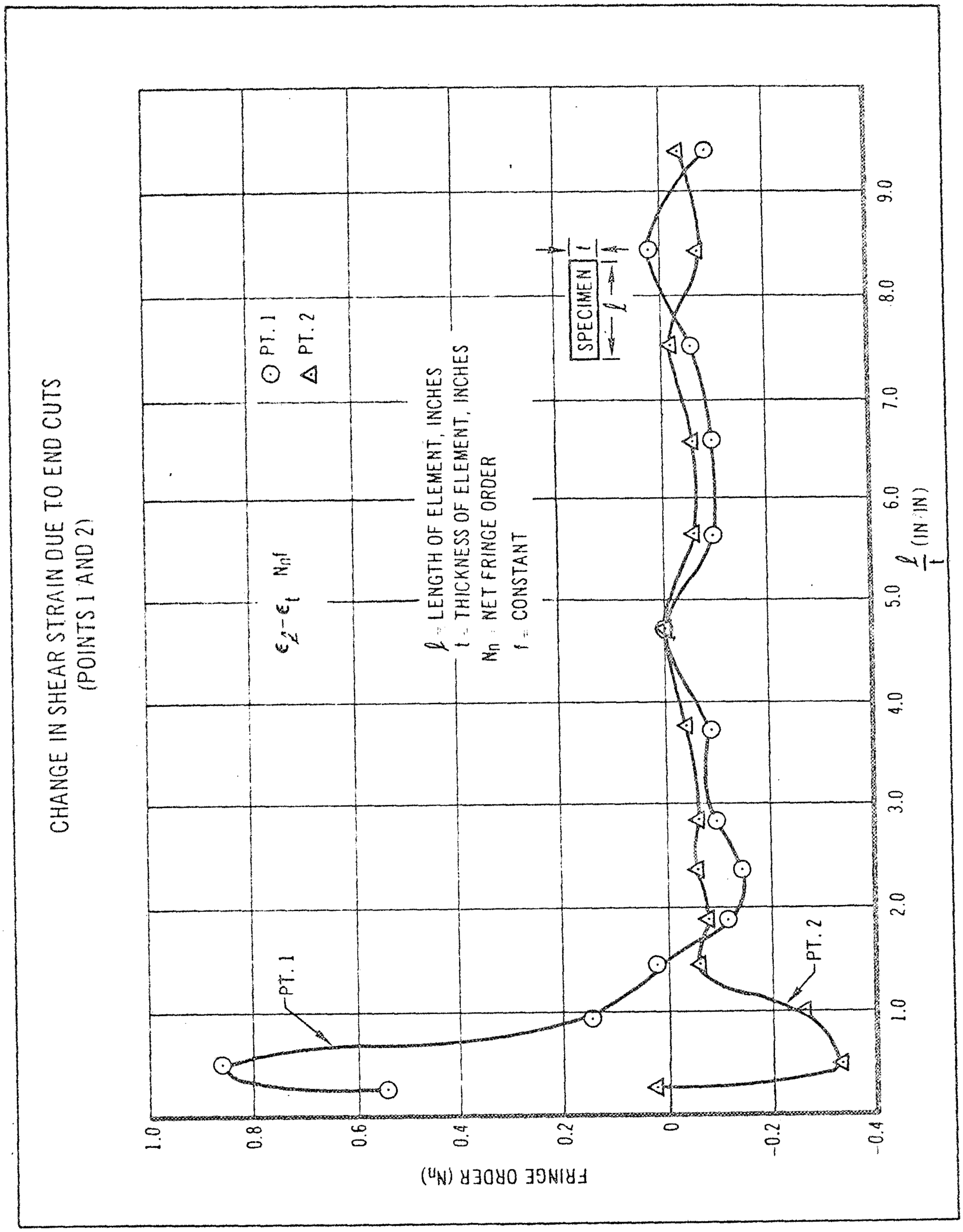

CHANGE IN SHEAR STRAIN DUE TO END CUTS
(POINTS 1 AND 2)
$\epsilon_{\ell} - \epsilon_{t} = N_n f$
ℓ = LENGTH OF ELEMENT, INCHES
t = THICKNESS OF ELEMENT, INCHES
N_n = NET FRINGE ORDER
f = CONSTANT
PT. 1
PT. 2
SPECIMEN
FRINGE ORDER (N_n)
$\frac{\ell}{t}$ (IN/IN)
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0

FIGURE 6-4

CHANGE IN SHEAR STRAIN DUE TO END CUTS
(POINTS 3, 4 AND 5)

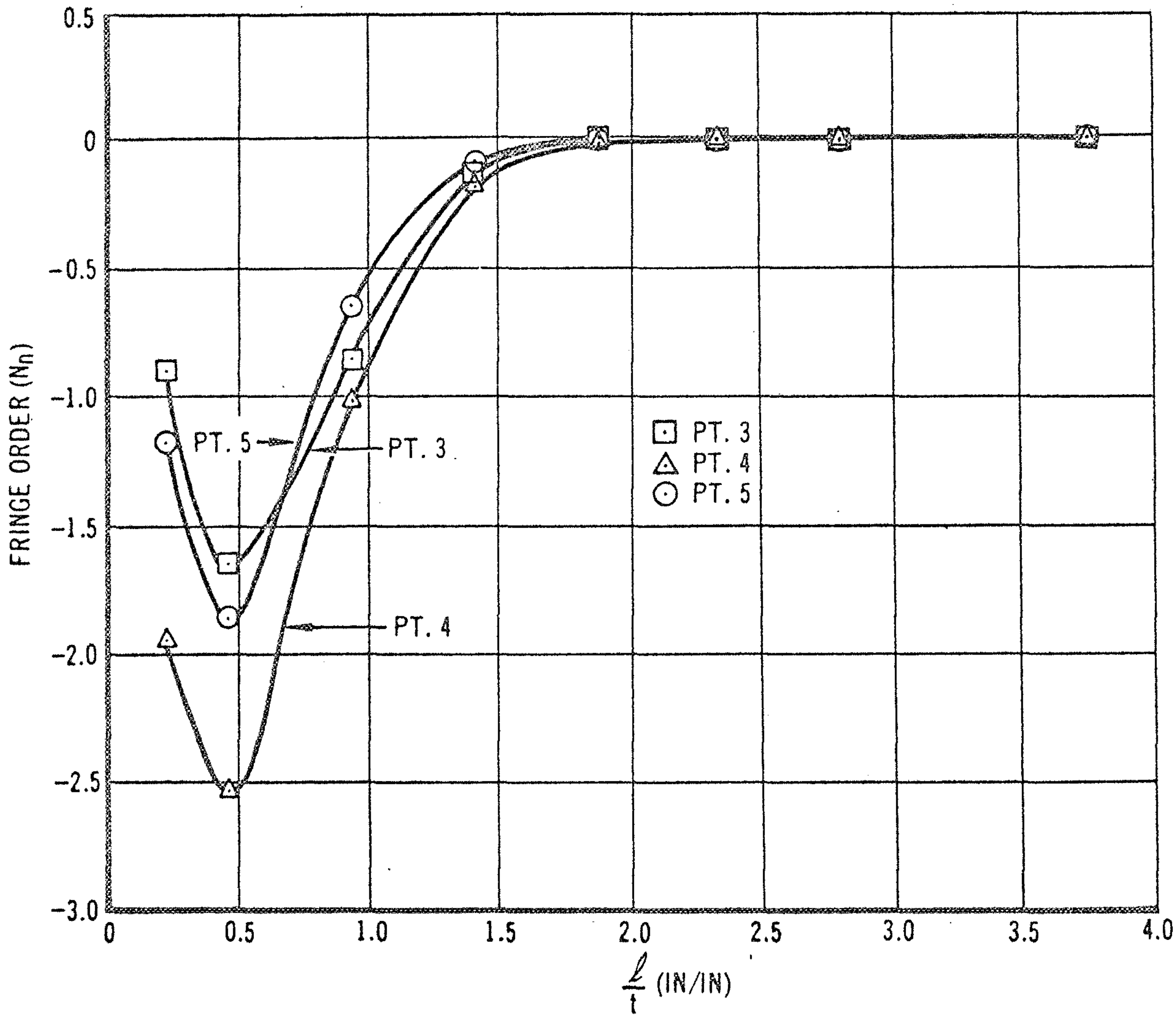


FIGURE 6-5

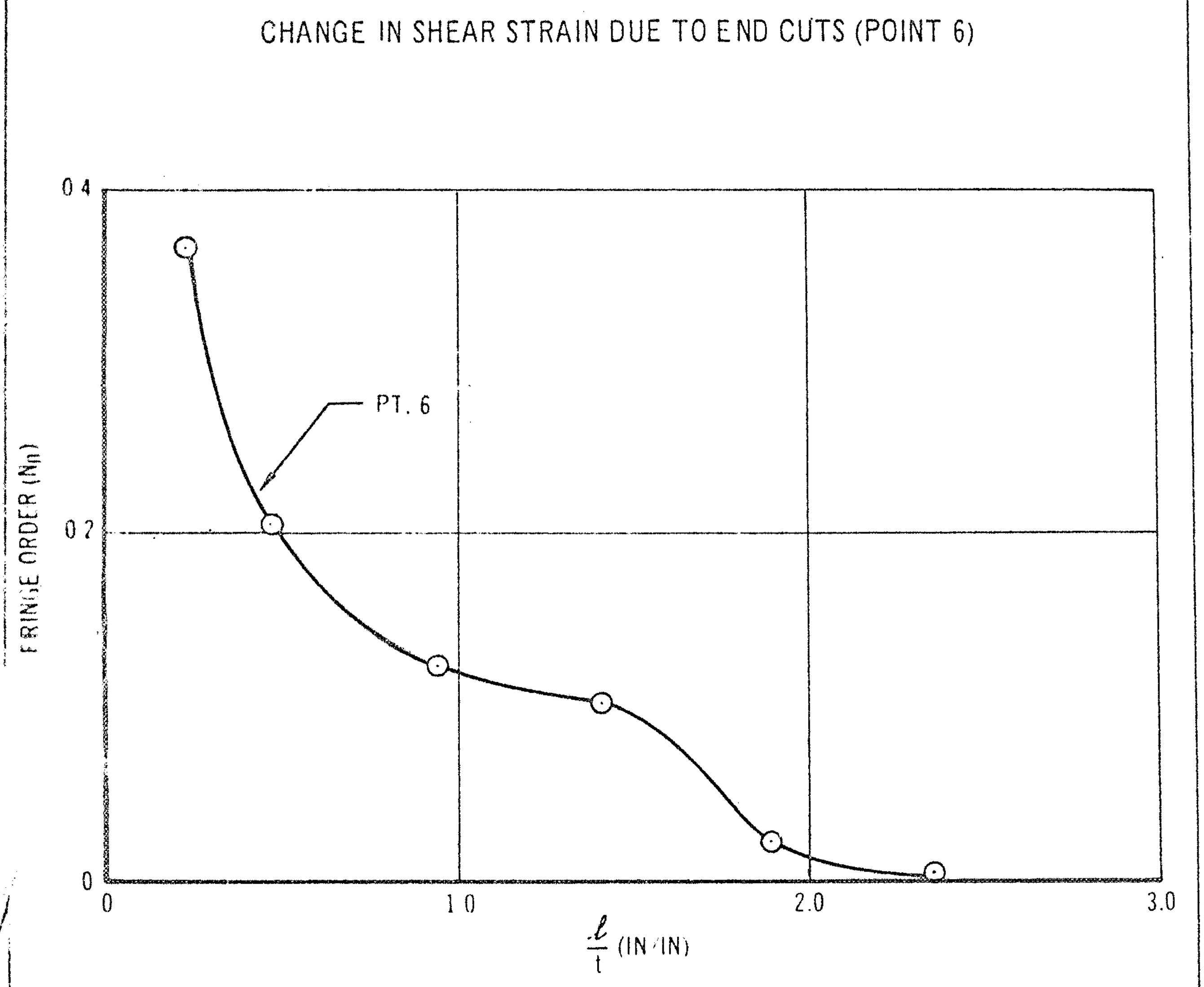

CHANGE IN SHEAR STRAIN DUE TO END CUTS (POINT 6)
0.4
0.2
0
FRINGE ORDER (Nn)
PT. 6
0
1.0
2.0
3.0
ℓ/t (IN/IN)

FIGURE 6-6

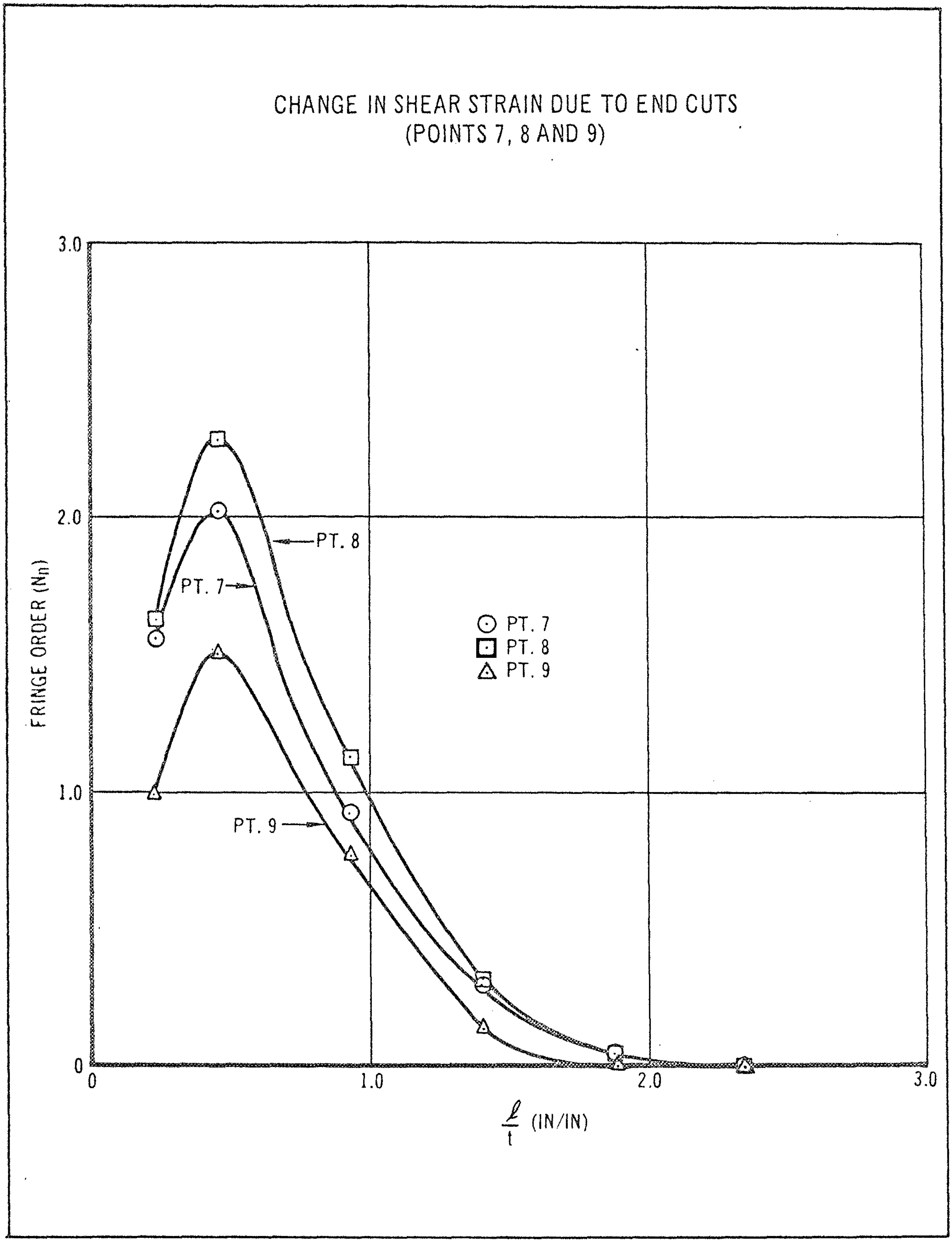

CHANGE IN SHEAR STRAIN DUE TO END CUTS
(POINTS 7, 8 AND 9)
3.0
2.0
1.0
0
FRINGE ORDER (N_n)
PT. 8
PT. 7
PT. 9
PT. 7
PT. 8
PT. 9
0
1.0
2.0
3.0
$\frac{\ell}{t}$ (IN/IN)

FIGURE 6-7

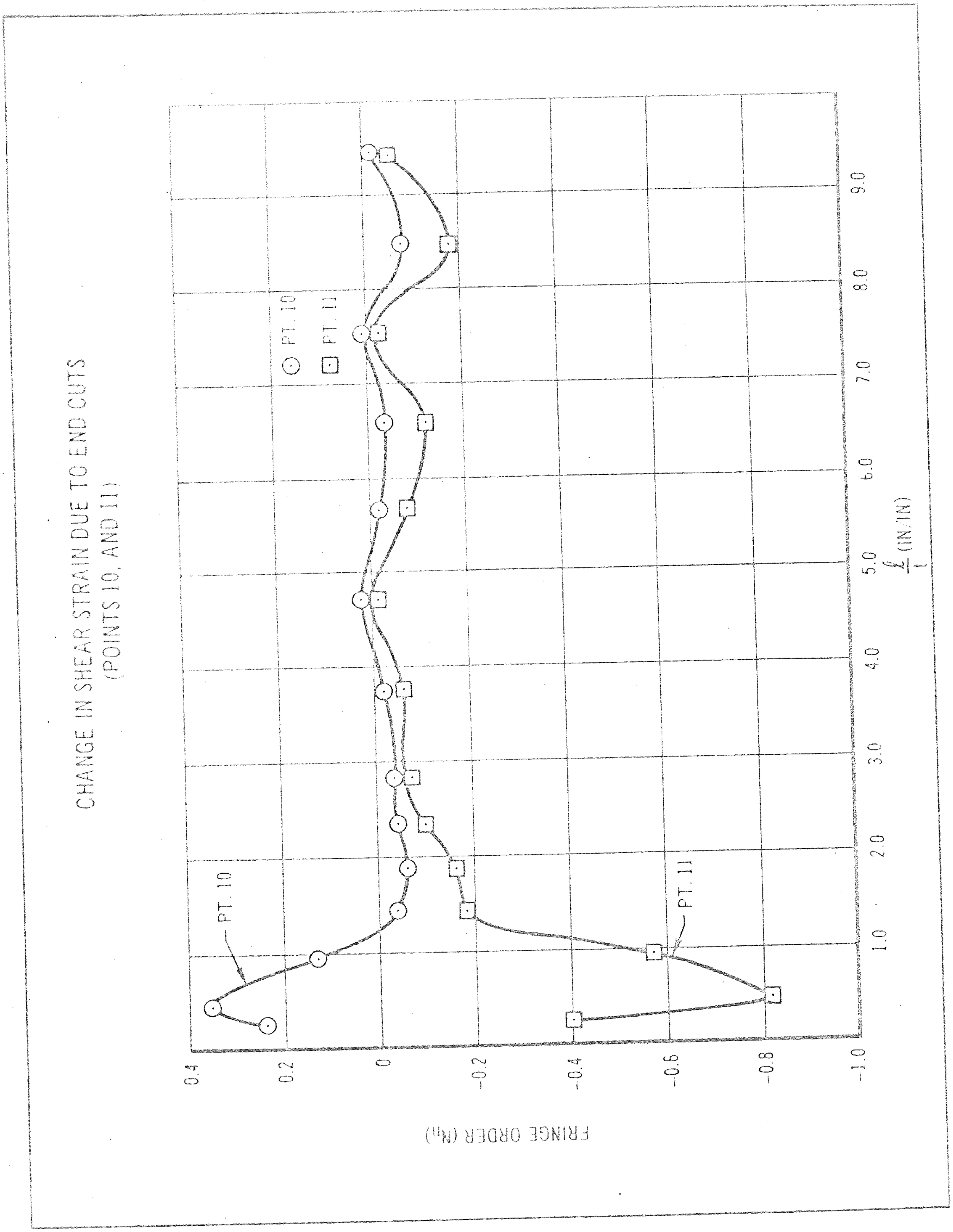

CHANGE IN SHEAR STRAIN DUE TO END CUTS
(POINTS 10, AND 11)
PT. 10
PT. 11
FRINGE ORDER (N_n)
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
$\frac{\ell}{t}$ (IN/IN)

FIGURE 6-8

6.2 Strain Gage Method

A second series of tests verified this conclusion. In these tests, strain gages rather then photostress material were applied to an aluminum specimen in order to determine dimensional changes. The specimen was a 2014-T6 aluminum bar 3 x 1 x 27 in., and was subjected to a bending moment of 115,700 in-lbs in a jig similar to that depicted in Figure 6-9. A drawing of the specimen, along with the type and location of the strain gages is shown in Figure 6-10.

Biaxial strain gages were then bonded to the top and bottom surfaces of the bar. Two of these were located on the specimens center line, and two were displaced a distance of 1-1/2 inches from the center line. An eight inch element was removed from the specimen and, following a procedure very similar to that used in the photostress tests, slices were progressively removed from each end of the element. All strain gages were read following the removal of each slice. A photo of the specimen taken during the slice removal process is shown in Figure 6-11. The data obtained from the center line gages (which was typical of the offset gages) is presented in Figures 6-12 and 6-13. As can be seen, these results were essentially identical with those obtained from the photostress tests i.e., a specimen length to thickness ratio of 2 is sufficient to insure that the strain distribution at the specimens center line remains unchanged by slice removal.

JIG USED TO BEND STRAIN GAGE SPECIMEN

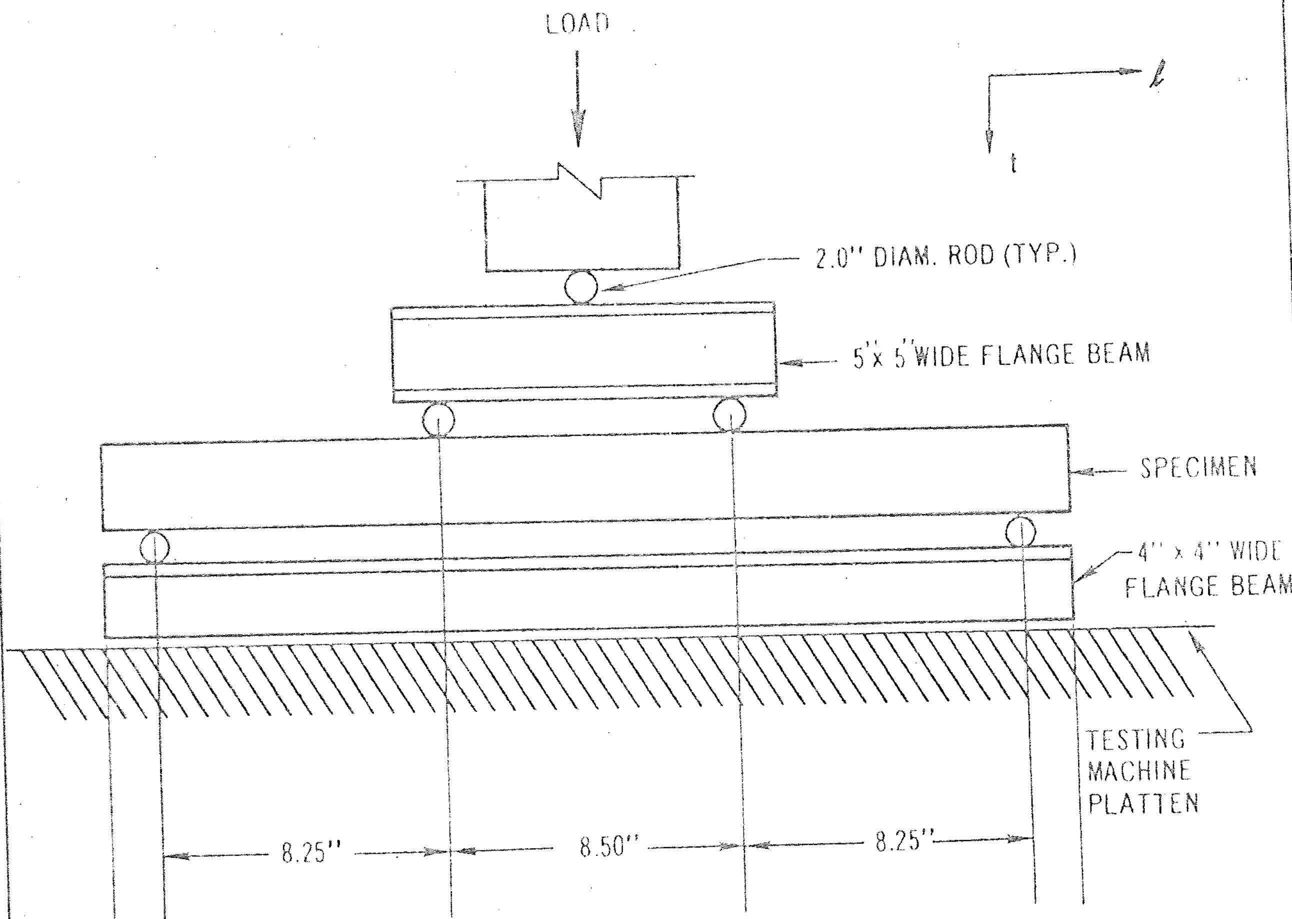
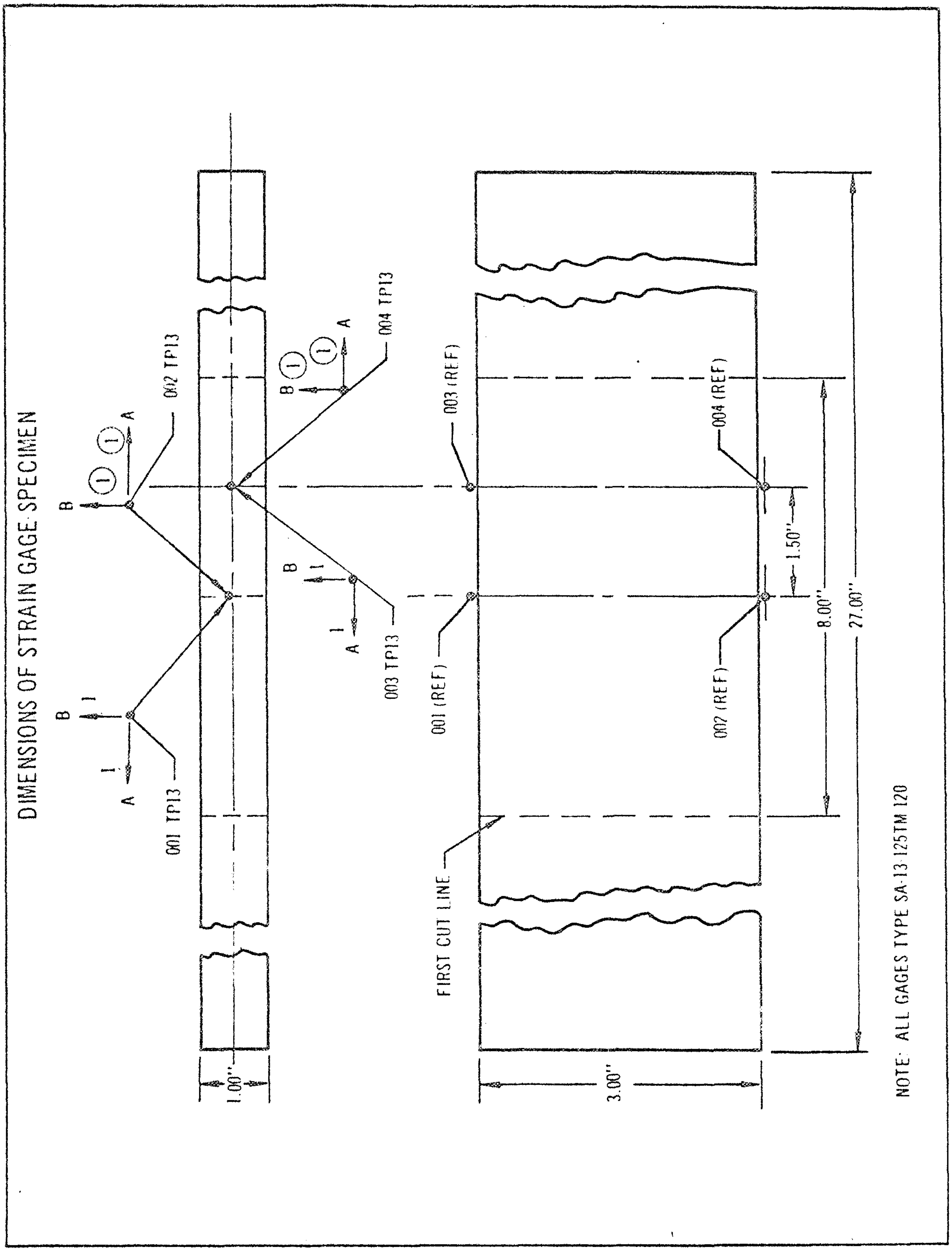
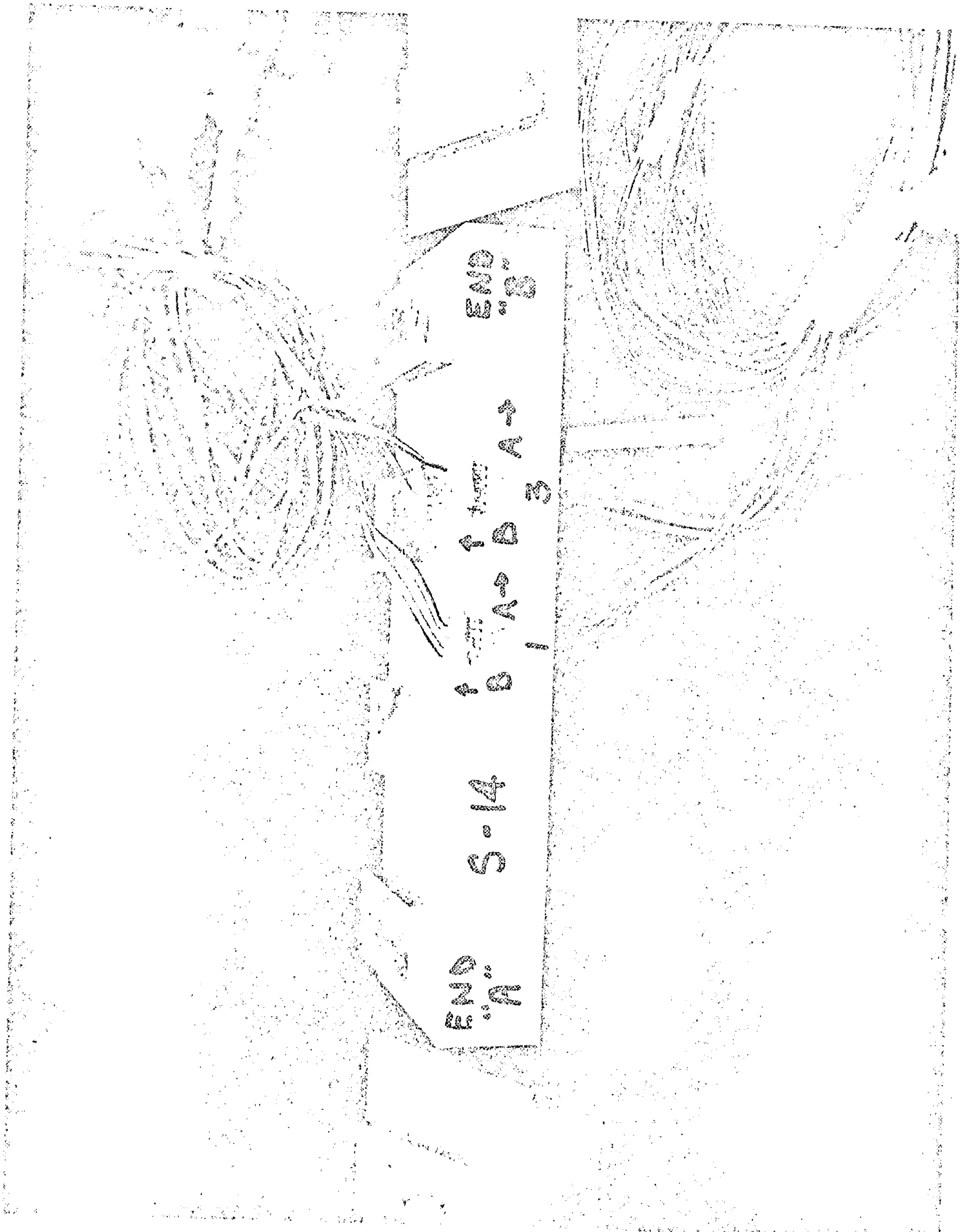


FIGURE 6-9



DIMENSIONS OF STRAIN GAGE SPECIMEN
001 TP13
002 TP13
003 TP13
004 TP13
A
B
1
001 (REF)
002 (REF)
003 (REF)
004 (REF)
FIRST CUT LINE
1.00"
3.00"
1.50"
8.00"
27.00"
NOTE: ALL GAGES TYPE SA-13-125TM 120

FIGURE 6-10



SPECIMEN DURING SLICE REMOVAL PROCESS

FIGURE 6-11

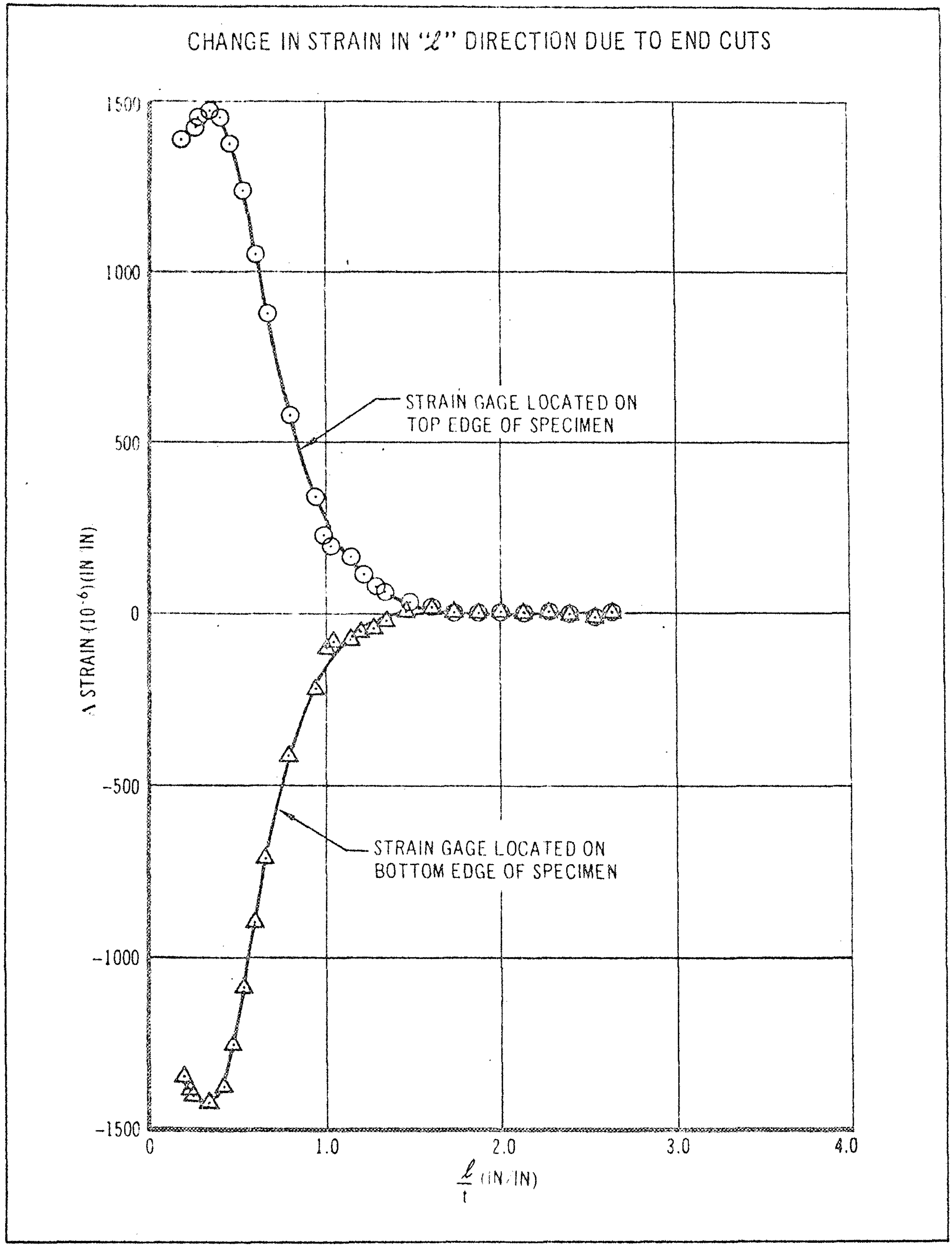

CHANGE IN STRAIN IN "ℓ" DIRECTION DUE TO END CUTS
1500
1000
500
0
-500
-1000
-1500
Δ STRAIN (10⁻⁶)(IN/IN)
STRAIN GAGE LOCATED ON
TOP EDGE OF SPECIMEN
STRAIN GAGE LOCATED ON
BOTTOM EDGE OF SPECIMEN
0
1.0
2.0
3.0
4.0
ℓ/t (IN/IN)

FIGURE 6-12

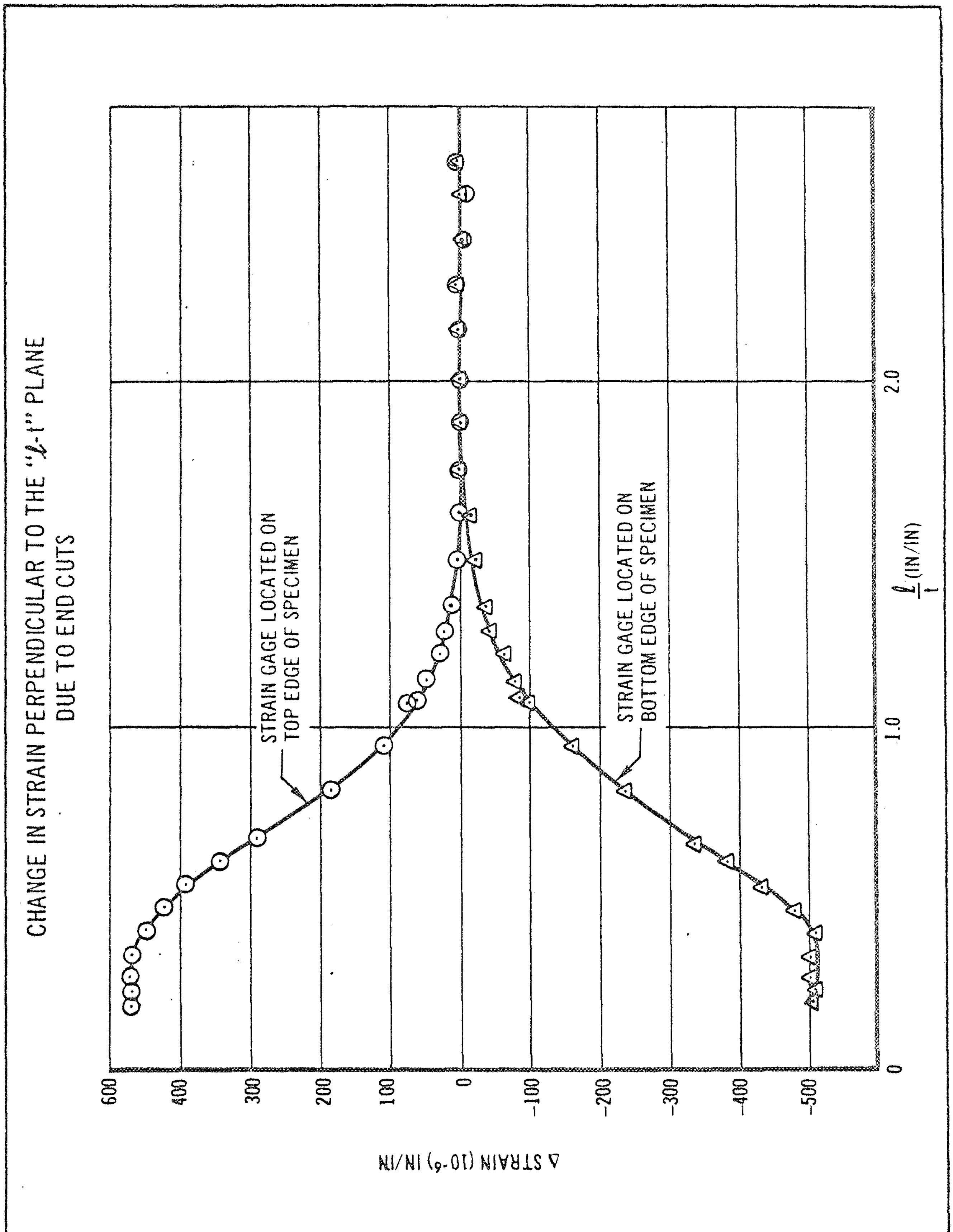

CHANGE IN STRAIN PERPENDICULAR TO THE "ℓ-t" PLANE
DUE TO END CUTS
STRAIN GAGE LOCATED ON
TOP EDGE OF SPECIMEN
STRAIN GAGE LOCATED ON
BOTTOM EDGE OF SPECIMEN
Δ STRAIN (10-6) IN/IN
600
400
300
200
100
0
-100
-200
-300
-400
-500
ℓ/t (IN/IN)
0
1.0
2.0

FIGURE 6-13

7. EXPERIMENTAL VERIFICATION PROGRAM

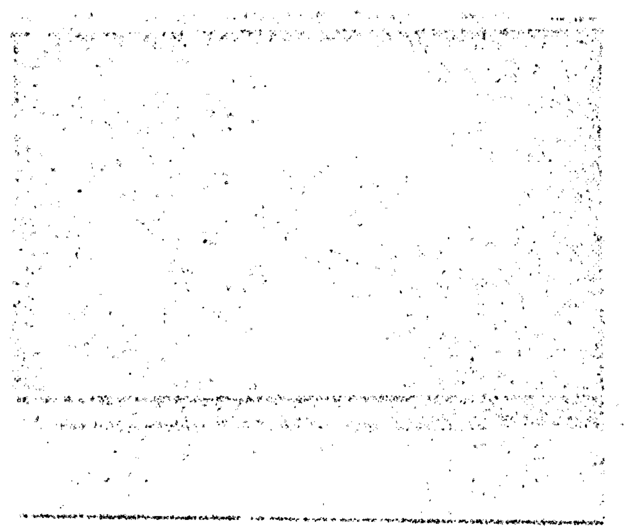
An experimental program was undertaken in order to determine the validity of the analysis procedure and the Fortran IV program. The experiments utilized aluminum beams of rectangular cross section which had been plastically deformed. The complete program is described in detail in the following paragraphs.

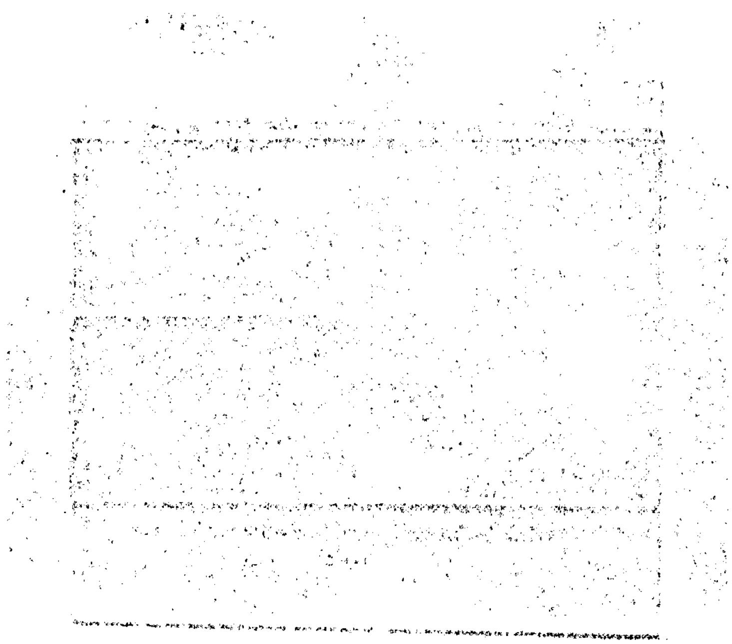
7.1 Experimental Procedure

Three 2014-T6 aluminum alloy bars 1 x 3 x 27-1/2 in. were stretched uniaxially in a tensile testing machine to a strain of 0.008 in/in. This strain was sufficient to cause yielding without any significant permanent changes in the bars cross sectional areas. Since the stress-strain curve of 2014-T6 shows relatively little strain hardening in its plastic range, this procedure relieved the bars of any extraneous residual stresses.

A three inch central section was then cut from one of the bars (Specimen 1) and one face was coated with photostress material. The 3 x 3 x 1 in. section was then sliced with a DoAll band saw and the slices inspected. The photostress coating indicated that no residual stresses were introduced due to the sawing operation (see Figure 7-1). The two remaining bars, Specimen 2 and 3, were then scribed with layout lines to facilitate the placement of the strain gages, and to act as guides for the subsequent slicing of the specimens. The bars were then subjected to a uniform bending moment. sufficient to cause plastic deformation. The loading arrangement used to bend the bars is depicted in Figure 6-9. A moment of 157,000 in. lb. was applied to specimen 2 and 148,500 in/lbs to specimen 3.

After the bars were bent, they were instrumented with strain gages. The types and locations of the gages, along with pertinent specimen dimensions are shown in Figures 7-2 and 7-3. Photographs of the instrumented specimens are shown in Figures 7-4 and 7-5. The two bars differed as to the number and locations of their longitudinal strain gages, and the magnitude of the bending moments to which they were subjected, but were identical in all other respects.

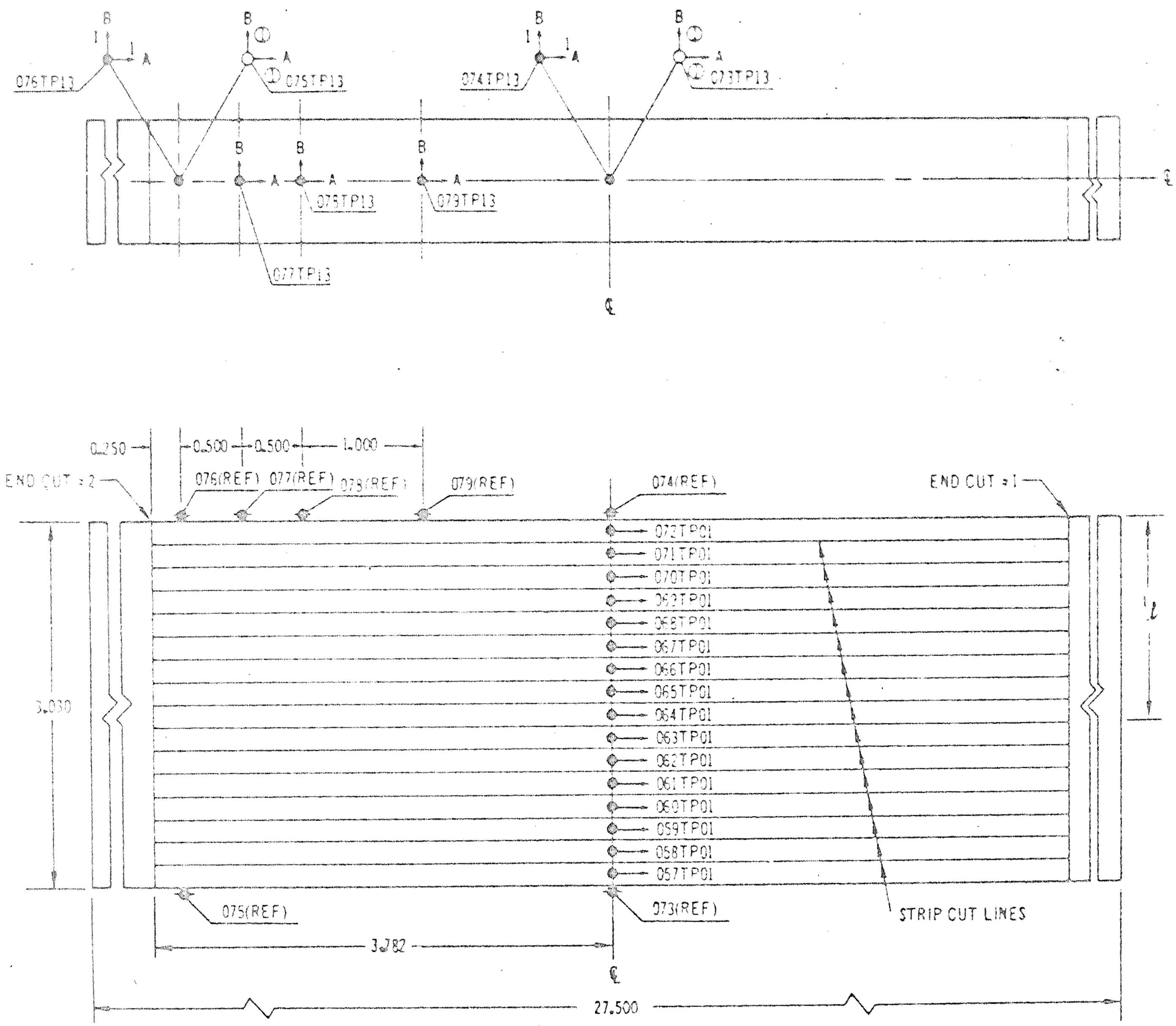




SPECIMEN 1 BEFORE AND AFTER SAWING WITH DOALL BAND SAW

FIGURE 7-1

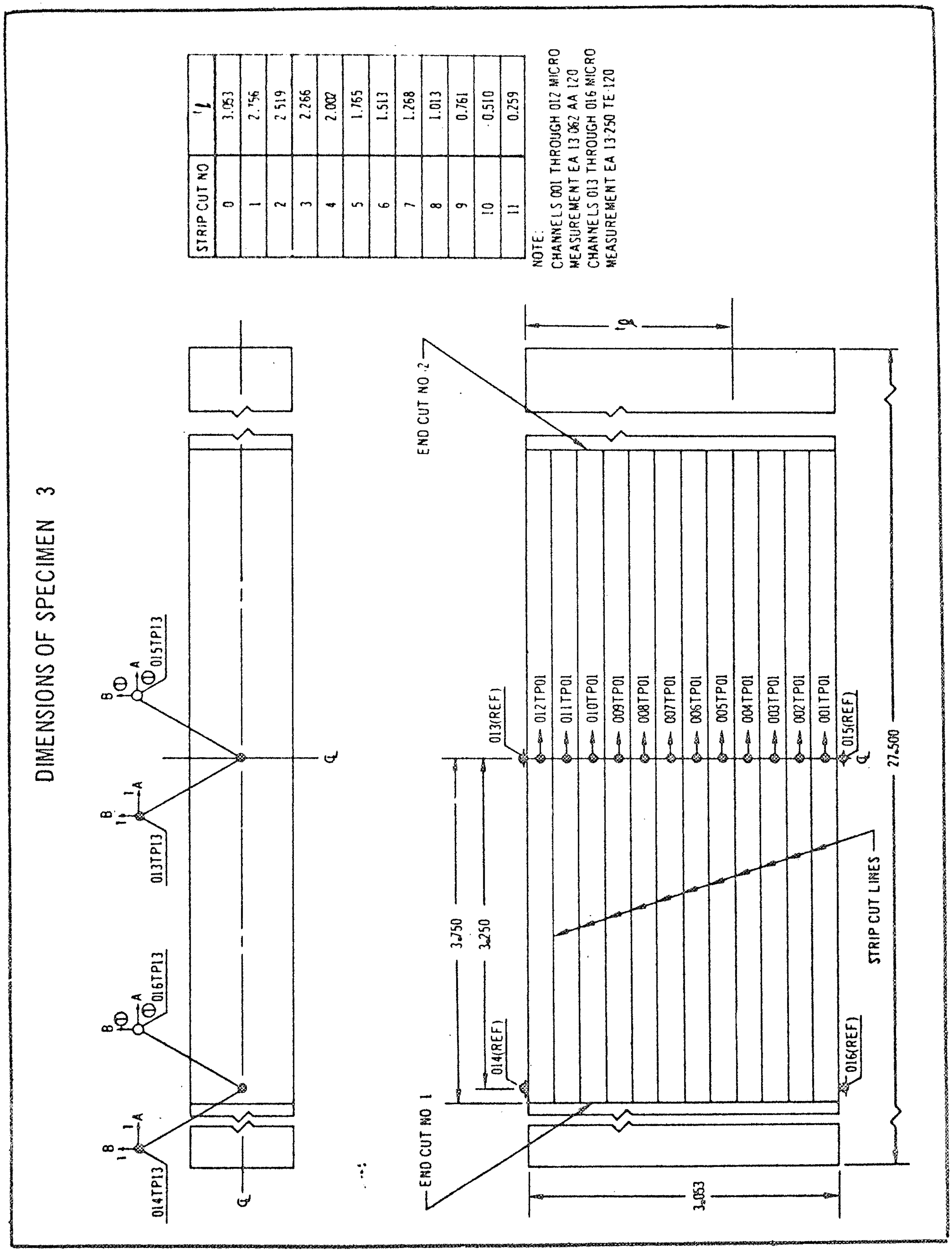
DIMENSIONS OF SPECIMEN 2



STRIP CUT NO	ℓ
0	3.030
1	2.829
2	2.639
3	2.455
4	2.268
5	2.075
6	1.865
7	1.705
8	1.513
9	1.307
10	1.121
11	0.919
12	0.728
13	0.526
14	0.356
15	0.133

NOTE.
CHANNELS 057 THROUGH 072 MICRO MEASUREMENT SA 13-013 DE-120
CHANNELS 073 THROUGH 079 MICRO MEASUREMENT SA-13-125 TM-120

FIGURE 7-2

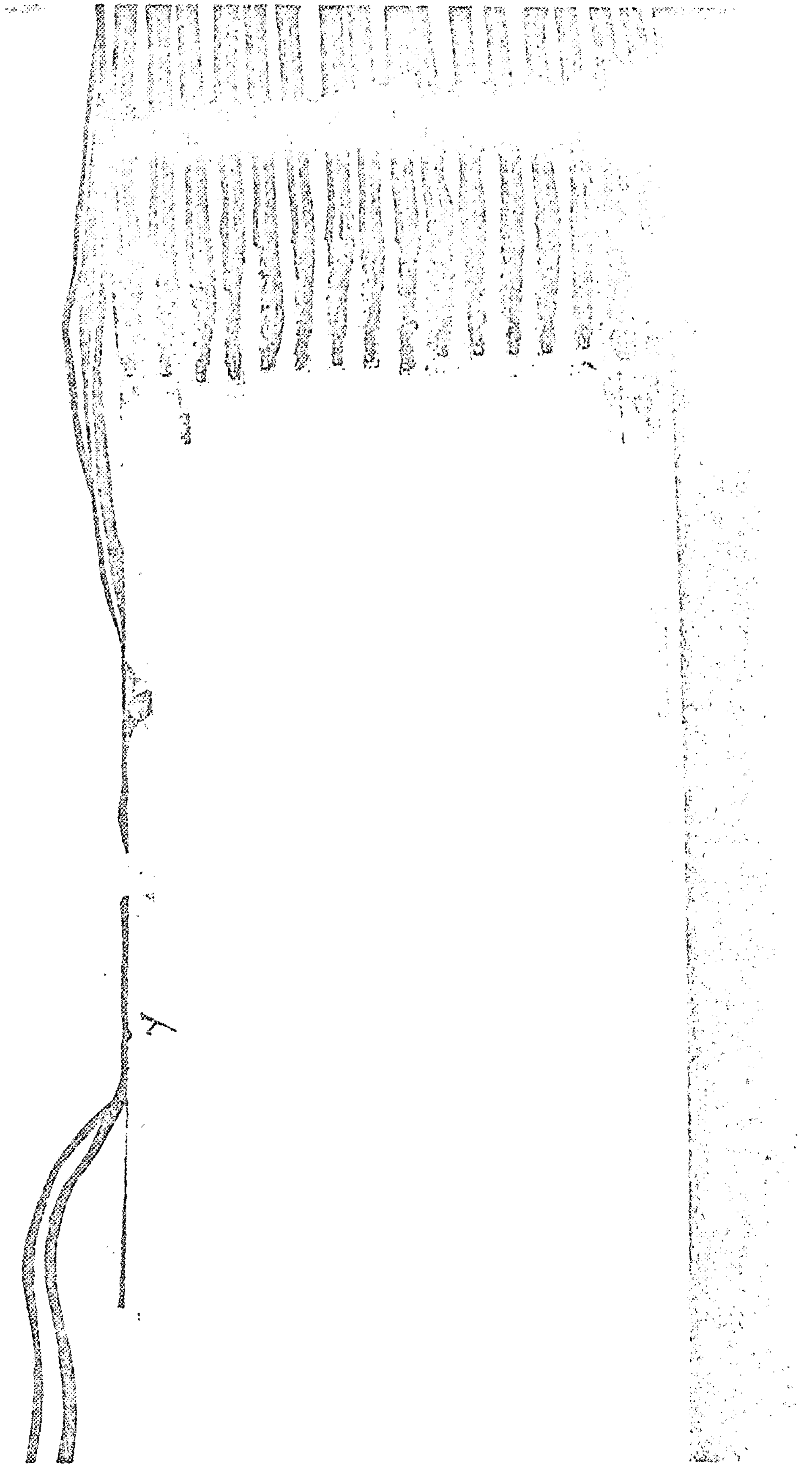


STRIP CUT NO	l_1
0	3.053
1	2.756
2	2.519
3	2.266
4	2.002
5	1.765
6	1.513
7	1.268
8	1.013
9	0.761
10	0.510
11	0.259

FIGURE 7-3

CUT LINES AND INSTALLATION OF STRAIN GAGES ON SPECIMEN 2

FIGURE 7-4



CUT LINES AND INSTALLATION OF STRAIN GAGES ON SPECIMEN 3

FIGURE 7-5

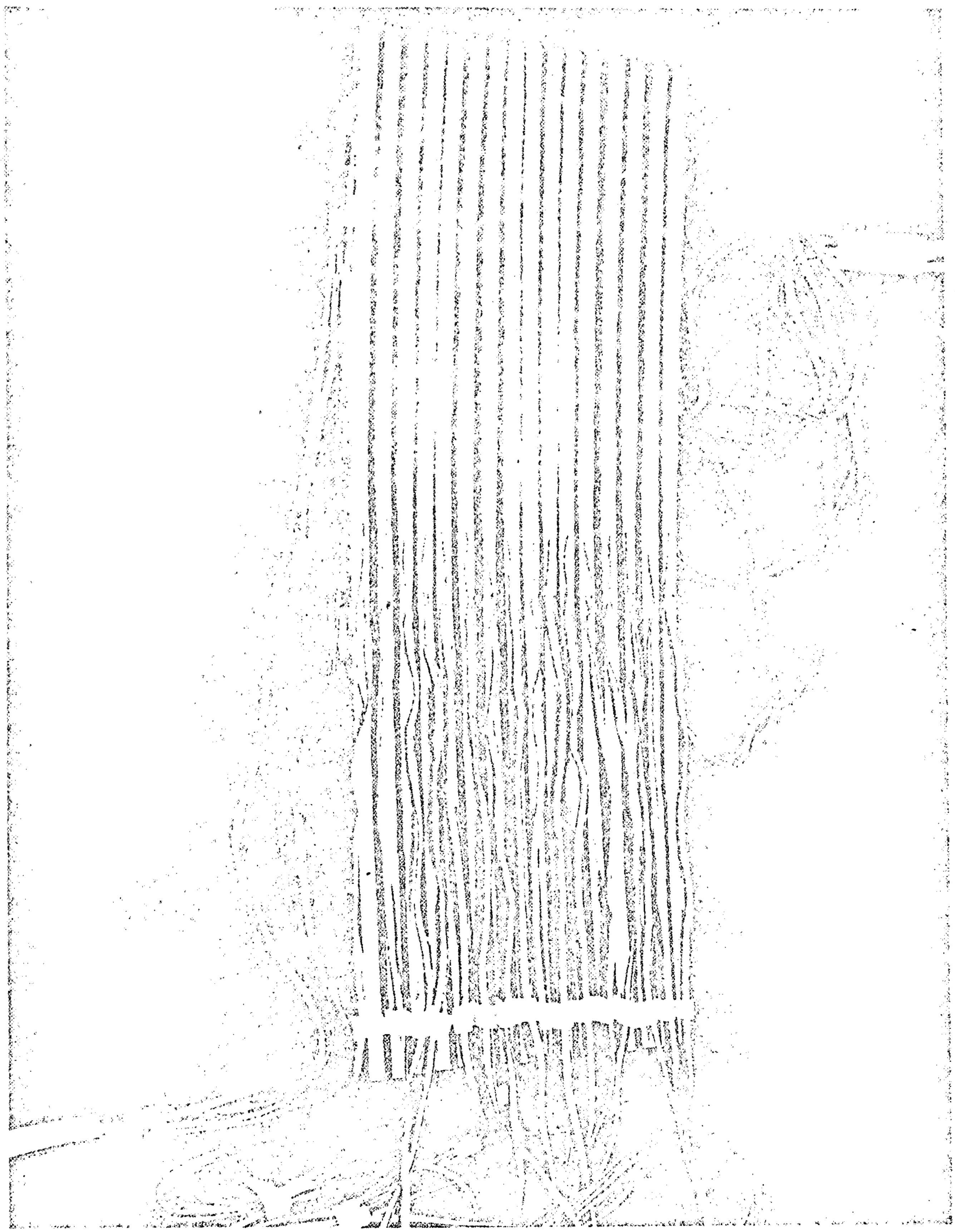
After instrumenting, a central section, 7-1/2 inches long, was removed from each bar. Taking care not to damage the strain gage leads, these 1 x 3 x 7-1/2 in. specimens were then sliced along the layout lines which were now arcs of a circle. Twelve slices were taken from specimen 2 and sixteen from 3. Photographs of the sliced specimens are shown in Figures 7-6 and 7-7. The large amount of strain relief experienced by each slice is clearly evident in these photographs. After the initial 7-1/2 in. cut, and after each slice, each strain gage was read with a Budd Co., Model P-350, portable strain indicator and the data recorded. A typical normalized data sheet is shown in Table 7-1.

7.2 Results

As can be seen from Table 7-1, the removal of the 7-1/2 in. central section, called out as end cut No. 1 and 2, had negligible effect on the strain gage readings. A residual stress profile in the longitudinal (X) direction was constructed by multiplying the released strain in each individual slice by the elastic modulus of the material. This stress was assumed to act at the center of each slice. The stresses in the Y and Z directions were assumed negligibly small, and subsequent data verified this hypothesis. A second residual stress profile was obtained using data from the specimens topmost strain gage in conjunction with the computer program. The resultant stress profiles for both specimens are depicted in Figures 7-8 and 7-9. As can be readily seen, correlation between the two is excellent.

SPECIMEN 2 AFTER SLICING

FIGURE 7-6



SPECIMEN 3 AFTER SLICING

FIGURE 7-7

TABL[illegible]7-1
NORMALIZED DATA SHEET

Δ STRAIN READINGS X (10^{-6}) ($\frac{IN}{IN}$)
FOR SPECIMEN NO. 2

GAGE NO.	AMBIENT ZERO	END CUT NO. 1	END CUT NO. 2	STRIP NO. 1 CUT	STRIP NO. 2 CUT	STRIP NO. 3 CUT	STRIP NO. 4 CUT	STRIP NO. 5 CUT	STRIP NO. 6 CUT	STRIP NO. 7 CUT	STRIP NO. 8 CUT	STRIP NO. 9 CUT	STRIP NO. 10 CUT	STRIP NO. 11 CUT
1	0	-7	22	-985										
2	0	-12	9	478	-310									
3	0	-9	17	447	691	498								
4	0	-15	4	359	560	535	1634							
5	0	-21	-2	282	433	415	-270	1617						
6	0	-10	13	226	341	312	-261	-203	254					
7	0	-8	7	154	244	213	-217	-994	-1764	-984				
8	6	1	11	70	129	109	-175	-700	-1228	-1761	-1399			
9	0	-4	15	33	21	10	-126	-409	-709	-1072	-1307	-1323		
10	0	6	17	-39	-92	-102	-80	-123	-219	-412	-539	-543	-568	
11	0	5	20	-104	-192	-213	-35	152	262	227	220	230	242	188
12	0	-6	20	-172	-304	-325	12	407	750	871	989	968	932	949
13A	0	24	42	-201	-359	-377	48	595	1105	1373	1565	1520	1483	1421
13B	0	8	25	126	160	132	7	-157	-294	-427	-454	-479	-469	-515
14A	0	13	106	-104	-292	-371	-35	516	998	1388	1483	1403	1372	1388
14B	0	19	-37	47	82	75	-10	-144	-314	-423	-524	-470	-437	-398
15A	0	25	47	-1395										
15B	0	31	52	615										
16A	0	29	41	-1320										
16B	0	29	39	550										

NOTE: GAGES NO. 13 THROUGH 16 ARE BIAXIAL

CORRELATION BETWEEN COMPUTER PROGRAM AND INDIVIDUAL STRAIN GAGE DATA, SPECIMEN 2

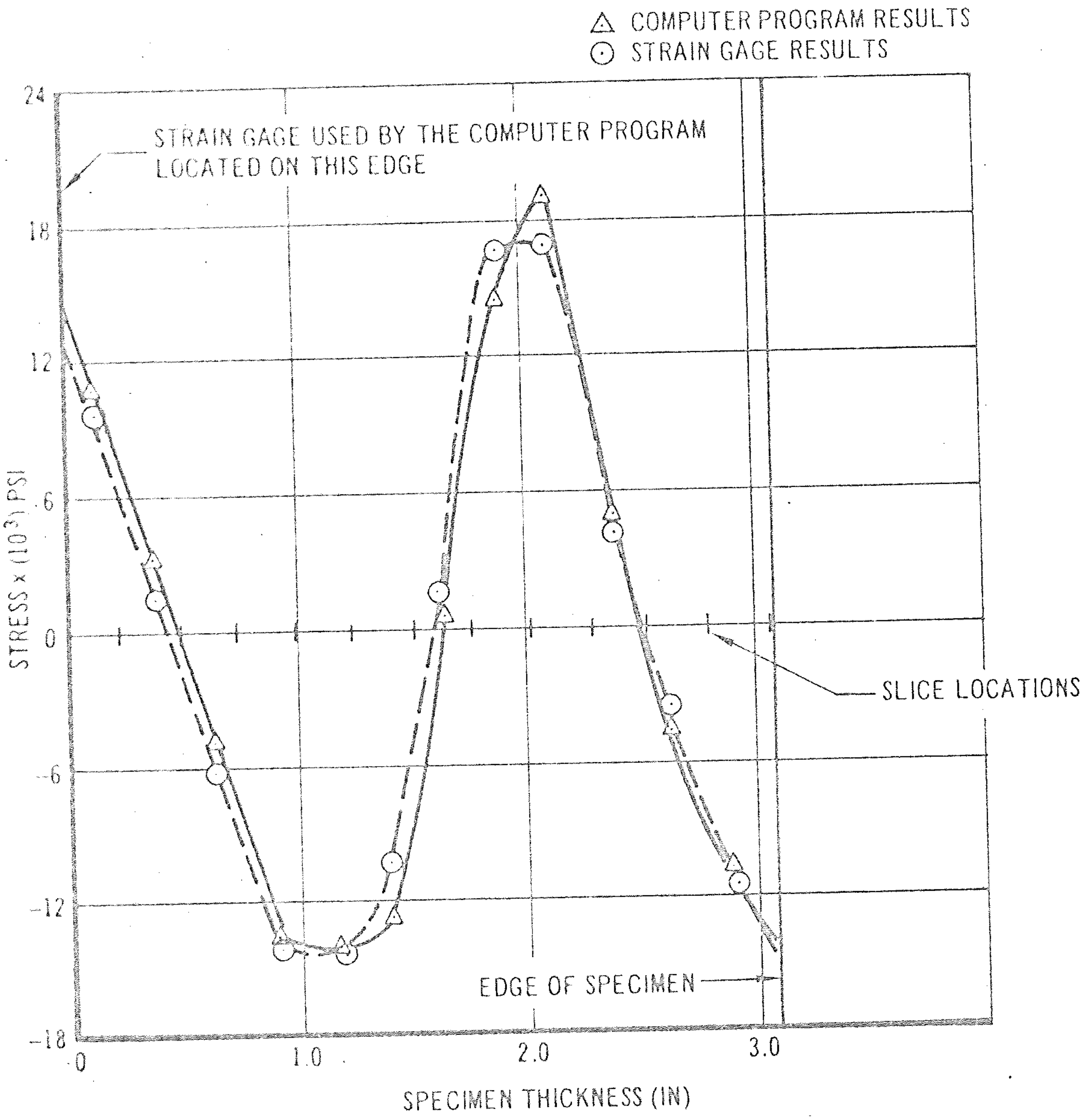


FIGURE 7-8

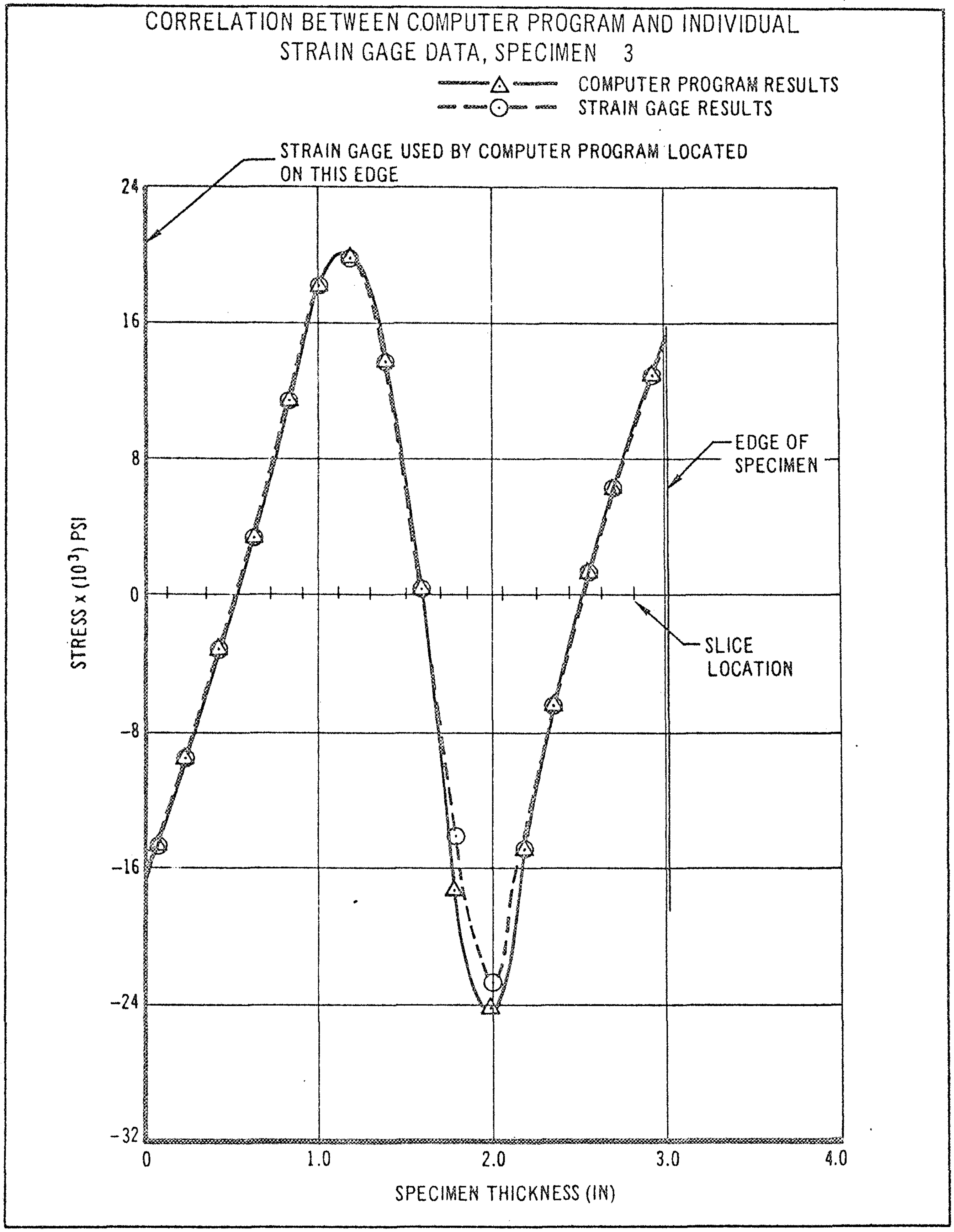

CORRELATION BETWEEN COMPUTER PROGRAM AND INDIVIDUAL
STRAIN GAGE DATA, SPECIMEN 3
COMPUTER PROGRAM RESULTS
STRAIN GAGE RESULTS
STRAIN GAGE USED BY COMPUTER PROGRAM LOCATED ON THIS EDGE
EDGE OF SPECIMEN
SLICE LOCATION
STRESS x (10^3) PSI
24
16
8
0
-8
-16
-24
-32
0
1.0
2.0
3.0
4.0
SPECIMEN THICKNESS (IN)

FIGURE 7-9

8. CONCLUSION

In conclusion the following has been accomplished:

A simple, accurate, computerized technique has been developed, and experimentally verified, to compute, print, plot and draw a curve of the residual stress distribution in two orthogonal directions of an isotropic or orthotropic structure.

The layer removal method is used to determine the non-linear residual stress distribution. This requires a specimen whose length and width has been experimentally determined to be twice the thickness of the specimen plus the gage length of the strain gage.

A literature survey has been made of previous work done in the field of measuring residual stresses.

9. ACKNOWLEDGMENT

The authors wish to express their appreciation to the following people who contributed greatly to the success of this project.

Mr. H. D. Meriwether for his assistance in the development of the Fortran IV program described in Paragraph 5, and for his valuable contributions to the test programs outlined in Paragraphs 6 and 7.

Mr. R. J. Schlamp for his technical advice and help in obtaining the proper use of, installation of, and data from the strain gages mentioned in Paragraphs 6 and 7.

Mr. R. W. McIver and Mr. R. D. Kerchenfaut for installing, reading and reporting the photo-stress data mentioned in Paragraph 6.

Mr. A. Rosner for his meticulous care and ability in machining the specimens mentioned in Paragraphs 6 and 7.

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DECEMBER [illegible], 1965

INQUIRY BY DOUGLAS AIRCRAFT CORP. ([illegible])
REGARDING INTEREST BY MSFC FOR STUDY OF:

"DETERMINATION OF RESIDUAL STRESSES
IN WELDS USING MATERIAL REMOVAL TECHNIQUES"

[illegible]

Welding of aluminum is used extensively throughout the Saturn vehicle and has achieved a high level. Further advances in the state-of-the-art will result from understanding of other weld characteristics, a prime example of which is residual stresses.

[illegible]

After review of the available research techniques throughout the industry, DAC derived and verified by experimentation an optimal analytical tool for establishing residual stresses (Reference 1). This method makes use of the variations in strains on one surface resulting from removal of stressed material from the other surface. Strain measurements are fed directly into a computer program which plots out the residual stresses which existed in the original coupon.

WORK TO DATE

DAC has run a limited research program to establish the magnitudes of residual stresses in welded flat plates of 0.1 inches thick (Reference 2). Data accruing from this study indicate values of residual stresses from clamping, welding and shaving for a spectrum of positions along and normal to the weldments. A significant example of the data already obtained is that DAC found that the heat cure cycles did not significantly effect the magnitudes of the residual stresses.

SUGGESTED RESEARCH PROGRAM

Work to date was on flat plates of 0.1 inches thick for various cure cycles. DAC investigated the variables and proposes that the next logical step would be investigation of the effect of material thickness, curvature and the effect

of structural proof tests on residual stresses.

These three parameters are of significant importance in the understanding of the properties of welds and results from this research program would aid in the advancement of the state-of-the-art in welding.

SUMMARY

Note:

A. DAC has a proven, simple and direct technique for determining residual stresses.

B. DAC proposes that three parameters be investigated:
The effect of thickness, curvature and proof tests on residual stresses.

REFERENCES

A. SM-46493, Revision 1, "Experimental Determination of Residual Stresses in an Orthotropic Material by the Use of Strain Gages and Material Removal," dated October 1965

B. SM-49077, "Residual Stresses in Mig Welds, dated October 1965

C. TWX A3-359-KOBA-4,1.7-T-75/W. M. Shempp to W. A. Mrazek, dated 10 November 1965

DAC CONTACT FOR FURTHER INFORMATION

W. M. Shempp, Ext. 3743 at DAC, Huntington Beach.

DECEMBER 2, 1968

INQUIRY BY DOUGLAS AIRCRAFT CORP. (F.B.)
REGARDING INTEREST BY MSFC FOR STUDY OF:

"DETERMINATION OF RESIDUAL STRESSES
IN WELDS USING MATERIAL REMOVAL TECHNIQUES"

PREMISE

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DAC CONTACT FOR FURTHER INFORMATION

W. M. Shempp, Ext. 3743 at DAC, Huntington Beach.

DECEMBER 2, 1969

INQUIRY BY DOUGLAS AIRCRAFT CORP. (H.B.)
REGARDING INTEREST BY MSFC FOR STUDY OF:

"DETERMINATION OF RESIDUAL STRESSES
IN WELDS USING MATERIAL REMOVAL TECHNIQUES"

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